

Vectors 11F

$$\begin{aligned}
 1 \text{ a Speed} &= |3\mathbf{i} + 4\mathbf{j}| \\
 &= \sqrt{3^2 + 4^2} \\
 &= \sqrt{9 + 16} \\
 &= \sqrt{25} \\
 &= 5 \text{ m s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{b Speed} &= |24\mathbf{i} + 7\mathbf{j}| \\
 &= \sqrt{24^2 + (-7)^2} \\
 &= \sqrt{576 + 49} \\
 &= \sqrt{625} \\
 &= 25 \text{ km h}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{c Speed} &= |5\mathbf{i} + 2\mathbf{j}| \\
 &= \sqrt{5^2 + 2^2} \\
 &= \sqrt{25 + 4} \\
 &= \sqrt{29} \\
 &= 5.39 \text{ m s}^{-1} \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{d Speed} &= |-7\mathbf{i} + 4\mathbf{j}| \\
 &= \sqrt{(-7)^2 + 4^2} \\
 &= \sqrt{49 + 16} \\
 &= \sqrt{65} \\
 &= 8.06 \text{ cm s}^{-1} \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ a Distance} &= \text{speed} \times \text{time} \\
 &= \sqrt{8^2 + 6^2} \times 5 \\
 &= 5 \times \sqrt{64 + 36} \\
 &= 5 \times \sqrt{100} \\
 &= 50 \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 \text{b Distance} &= \text{speed} \times \text{time} \\
 &= \sqrt{5^2 + (-1)^2} \times 10 \\
 &= 10 \times \sqrt{25 + 1} \\
 &= 10 \times \sqrt{26} \\
 &= 51.0 \text{ m (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ c Distance} &= \text{speed} \times \text{time} \\
 &= \sqrt{6^2 + 2^2} \times 0.75 \\
 &= 0.75 \times \sqrt{36 + 4} \\
 &= 0.75 \times \sqrt{40} \\
 &= 4.74 \text{ km (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{d Distance} &= \text{speed} \times \text{time} \\
 &= \sqrt{(-4)^2 + (-7)^2} \times 120 \\
 &= 120 \times \sqrt{16 + 49} \\
 &= 120 \times \sqrt{65} \\
 &= 967 \text{ cm (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ a Speed} &= \sqrt{(-3)^2 + 4^2} \\
 &= \sqrt{9 + 16} \\
 &= \sqrt{25} \\
 &= 5 \text{ m s}^{-1}
 \end{aligned}$$

$$\text{Distance} = 5 \times 15 = 75 \text{ m}$$

$$\begin{aligned}
 \text{b Speed} &= \sqrt{2^2 + 5^2} \\
 &= \sqrt{4 + 25} \\
 &= \sqrt{29} \\
 &= 5.39 \text{ m s}^{-1} \text{ (3 s.f.)}
 \end{aligned}$$

$$\text{Distance} = 3 \times 5.39 = 16.2 \text{ m (3 s.f.)}$$

$$\begin{aligned}
 \text{c Speed} &= \sqrt{5^2 + (-2)^2} \\
 &= \sqrt{25 + 4} \\
 &= \sqrt{29} \\
 &= 5.39 \text{ km h}^{-1} \text{ (3 s.f.)}
 \end{aligned}$$

$$\text{Distance} = 3 \times 5.39 = 16.2 \text{ km (3 s.f.)}$$

$$\begin{aligned}
 3 \text{ d Speed} &= \sqrt{12^2 + (-5)^2} \\
 &= \sqrt{144 + 25} \\
 &= \sqrt{169} \\
 &= 13 \text{ km h}^{-1}
 \end{aligned}$$

$$\text{Distance} = 0.5 \times 13 = 6.5 \text{ km}$$

$$\begin{aligned}
 4 \text{ a } \mathbf{a} &= \frac{(16\mathbf{i} - 5\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j})}{5} \\
 &= \frac{14\mathbf{i} - 8\mathbf{j}}{5} \\
 &= 2.8\mathbf{i} - 1.6\mathbf{j} \text{ m s}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 5 \text{ a } \tan \theta &= \frac{7}{5} \\
 \theta &= \tan^{-1} \frac{7}{5} \\
 &= 54.5^\circ \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b magnitude of } \mathbf{F} &= |\mathbf{ma}| \\
 &= 0.3\sqrt{5^2 + 7^2} \\
 &= 0.3\sqrt{74}
 \end{aligned}$$

$$\begin{aligned}
 6 \text{ a } \tan \theta &= \frac{1}{2} \\
 \theta &= \tan^{-1} \frac{1}{2} \\
 &= 26.6^\circ \text{ below}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } 3\mathbf{i} - 4\mathbf{j} + p\mathbf{i} + q\mathbf{j} &= \lambda(2\mathbf{i} - \mathbf{j}) \\
 (3 + p)\mathbf{i} + (q - 4)\mathbf{j} &= 2\lambda\mathbf{i} - \lambda\mathbf{j} \\
 3 + p &= 2\lambda \text{ and } q - 4 = -\lambda
 \end{aligned}$$

Multiplying the second equation by 2:

$$2q - 8 = -2\lambda$$

Solving simultaneously:

$$\begin{aligned}
 3 + p &= -2q + 8 \\
 p + 2q &= 5
 \end{aligned}$$

$$\begin{aligned}
 \text{c When } p = 1, \lambda = 2 \\
 \mathbf{R} &= 2(2\mathbf{i} - \mathbf{j}) \\
 &= 4\mathbf{i} - 2\mathbf{j}
 \end{aligned}$$

$$\begin{aligned}
 |\mathbf{R}| &= \sqrt{4^2 + 2^2} \\
 &= \sqrt{20} \\
 &= 2\sqrt{5} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 7 \text{ a } \overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AC} \\
 &= -(30\mathbf{i} + 40\mathbf{j}) + 40\mathbf{i} - 60\mathbf{j} \\
 &= 10\mathbf{i} - 100\mathbf{j}
 \end{aligned}$$

$$\begin{aligned}
 7 \text{ b } AB &= \sqrt{30^2 + 40^2} = \sqrt{2500} = 50 \\
 AC &= \sqrt{40^2 + (-60)^2} = \sqrt{5200} \\
 BC &= \sqrt{10^2 + (-100)^2} = \sqrt{10100}
 \end{aligned}$$

Using the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{\sqrt{5200}^2 + 50^2 - \sqrt{10100}^2}{2(\sqrt{5200})(50)}$$

$$\cos A = \frac{5200 + 2500 - 10100}{1000\sqrt{52}}$$

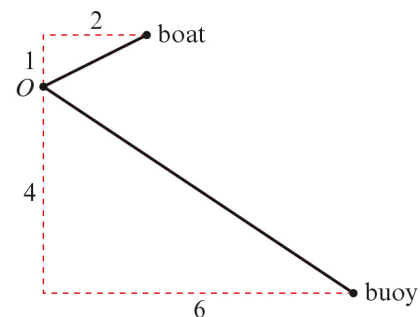
$$\cos A = \frac{-2400}{1000\sqrt{52}}$$

$$A = 109.44003\dots$$

So $\angle BAC = 109.4^\circ$

$$\begin{aligned}
 \text{c Area} &= \frac{1}{2}bc \sin A \\
 &= \frac{1}{2} \times \sqrt{5200} \times 50 \times \sin 109.4^\circ \\
 &= 1700.418\dots \\
 &= 1700 \text{ m}^2 \text{ (3 s.f.)}
 \end{aligned}$$

8 a



$$\begin{aligned}
 \text{Distance} &= \sqrt{(6-2)^2 + (-4-1)^2} \\
 &= \sqrt{16+25} \\
 &= \sqrt{41} \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 \text{b Bearing} &= 270^\circ + \tan^{-1} \frac{5}{4} \\
 &= 321.3^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{c The vector of the boat to the buoy} \\
 &= -(2\mathbf{i} + \mathbf{j}) + 6\mathbf{i} - 4\mathbf{j} = 4\mathbf{i} - 5\mathbf{j} \\
 \text{Velocity} &= 8\mathbf{i} - 10\mathbf{j} \\
 \text{so } \lambda(8\mathbf{i} - 10\mathbf{j}) &= 4\mathbf{i} - 5\mathbf{j} \\
 \lambda &= \frac{1}{2}
 \end{aligned}$$

Therefore, the boat is travelling directly towards the buoy.

$$\begin{aligned} \mathbf{8\ d} \quad \text{Speed} &= \sqrt{8^2 + 10^2} \\ &= \sqrt{164} \\ &= 2\sqrt{41} \text{ km h}^{-1} \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad \text{Time} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{\sqrt{41}}{2\sqrt{41}} \\ &= \frac{1}{2} \text{ hour} \\ &= 30 \text{ minutes} \end{aligned}$$