

Vectors 11D

1 a i  $\overline{OA} = 3\mathbf{i} - \mathbf{j}$ ,  $\overline{OB} = 4\mathbf{i} + 5\mathbf{j}$ ,  
 $\overline{OC} = -2\mathbf{i} + 6\mathbf{j}$

ii  $\overline{AB} = B - A$   
 $= (4\mathbf{i} + 5\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$   
 $= 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{i} + \mathbf{j}$   
 $= \mathbf{i} + 6\mathbf{j}$

iii  $\overline{AC} = C - A$   
 $= (-2\mathbf{i} + 6\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$   
 $= -2\mathbf{i} + 6\mathbf{j} - 3\mathbf{i} + \mathbf{j}$   
 $= -5\mathbf{i} + 7\mathbf{j}$

b i  $|\overline{OC}| = |-2\mathbf{i} + 6\mathbf{j}|$   
 $= \sqrt{(-2)^2 + 6^2}$   
 $= \sqrt{40}$   
 $= \sqrt{4}\sqrt{10}$   
 $= 2\sqrt{10}$

ii  $|\overline{AB}| = |\mathbf{i} + 6\mathbf{j}|$   
 $= \sqrt{1^2 + 6^2}$   
 $= \sqrt{37}$

iii  $|\overline{AC}| = |-5\mathbf{i} + 7\mathbf{j}|$   
 $= \sqrt{(-5)^2 + 7^2}$   
 $= \sqrt{74}$

2 a  $\overline{PQ} = -\overline{OP} + \overline{OQ}$   
 $= -(4\mathbf{i} - 3\mathbf{j}) + 3\mathbf{i} + 2\mathbf{j}$   
 $= -\mathbf{i} + 5\mathbf{j}$

or

$$-\begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

b i  $|\overline{OP}| = \sqrt{4^2 + (-3)^2}$   
 $= \sqrt{25} = 5$

ii  $|\overline{OQ}| = \sqrt{3^2 + 2^2}$   
 $= \sqrt{13}$

2 b iii  $|\overline{PQ}| = \sqrt{(-1)^2 + 5^2}$   
 $= \sqrt{26}$

3 a  $\overline{PQ} = -\overline{OP} + \overline{OQ}$   
 $= 5\mathbf{i} + 6\mathbf{j}$   
 $= -\overline{OP} + (4\mathbf{i} - 3\mathbf{j})$

$$-\overline{OP} = 5\mathbf{i} + 6\mathbf{j} - (4\mathbf{i} - 3\mathbf{j})$$

$$= \mathbf{i} + 9\mathbf{j}$$

$$\overline{OP} = -\mathbf{i} - 9\mathbf{j}$$

or

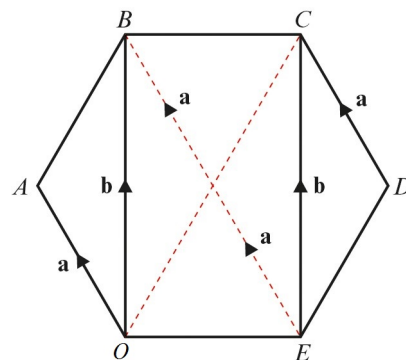
$$\overline{OP} = \begin{pmatrix} -1 \\ -9 \end{pmatrix}$$

b i  $|\overline{OP}| = \sqrt{(-1)^2 + (-9)^2}$   
 $= \sqrt{82}$

ii  $|\overline{OQ}| = \sqrt{4^2 + (-3)^2}$   
 $= \sqrt{25}$   
 $= 5$

iii  $|\overline{PQ}| = \sqrt{5^2 + 6^2}$   
 $= \sqrt{61}$

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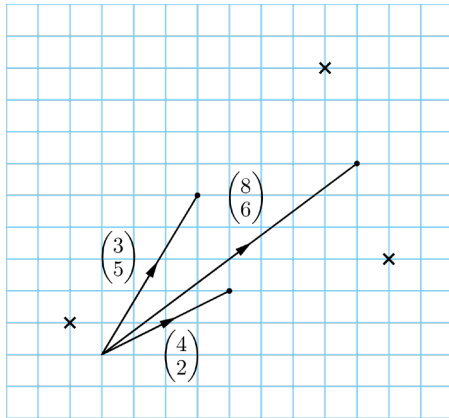


a  $\overline{OC} = \overline{OB} + \overline{BE} + \overline{EC}$   
 $= \mathbf{b} - 2\mathbf{a} + \mathbf{b}$   
 $= -2\mathbf{a} + 2\mathbf{b}$

b  $\overline{OD} = \overline{OC} + \overline{CD}$   
 $= -2\mathbf{a} + 2\mathbf{b} - \mathbf{a}$   
 $= -3\mathbf{a} + 2\mathbf{b}$

$$\begin{aligned}
 4 \text{ c } \quad \overrightarrow{OE} &= \overrightarrow{OB} + \overrightarrow{BE} \\
 &= \mathbf{b} - 2\mathbf{a} \\
 &= -2\mathbf{a} + \mathbf{b}
 \end{aligned}$$

5 The sketch shows the three possible positions of the fourth vertex.



$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

or

$$-\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

or

$$\begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

So  $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$  or  $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$  or  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$\begin{aligned}
 6 \text{ a } \quad \overrightarrow{AB} &= -(4\mathbf{i} - 5\mathbf{j}) + 6\mathbf{i} + 3\mathbf{j} \\
 &= 2\mathbf{i} + 8\mathbf{j}
 \end{aligned}$$

$$\begin{aligned}
 6 \text{ b } \quad |\overrightarrow{AB}| &= \sqrt{2^2 + 8^2} \\
 &= \sqrt{68} \\
 &= 2\sqrt{17}
 \end{aligned}$$

7 Using the radius of the circle

$$|\overrightarrow{OA}| = 3$$

Using the position vector

$$|\overrightarrow{OA}| = \sqrt{4k^2 + k^2}$$

$$= \sqrt{5k^2}$$

$$= \sqrt{5}k$$

$$\sqrt{5}k = 3$$

$$|k| = \frac{3}{\pm\sqrt{5}}$$

Rationalising the denominator

$$k = \pm \frac{3\sqrt{5}}{5}$$

### Challenge

Using Pythagoras' theorem

$$x^2 + y^2 = 13$$

Solve the equations simultaneously.

Substitute  $y = 6 - \frac{3}{2}x$  into  $x^2 + y^2 = 13$ :

$$x^2 + (6 - \frac{3}{2}x)^2 = 13$$

$$x^2 + 36 - 18x + \frac{9}{4}x^2 - 13 = 0$$

$$13x^2 - 72x + 92 = 0$$

$$(13x - 46)(x - 2) = 0$$

$$x = \frac{46}{13} \text{ or } x = 2$$

When  $x = \frac{46}{13}$ ,  $y = \frac{9}{13}$

When  $x = 2$ ,  $y = 3$

$$\overrightarrow{OB} = \frac{46}{13}\mathbf{i} + \frac{9}{13}\mathbf{j} \text{ or}$$

$$\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$$