Vectors 11C

1 a $|3\mathbf{i}+4\mathbf{j}|=\sqrt{3^2+4^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

b $|6\mathbf{i}-8\mathbf{j}|=\sqrt{6^2+8^2}$
$=\sqrt{36+64}$
$=\sqrt{100}$
$=10$

c $|5\mathbf{i}+12\mathbf{j}|=\sqrt{5^2+12^2}$
$=\sqrt{25+144}$
$=\sqrt{169}$
$=13$

d $|2\mathbf{i}+4\mathbf{j}|=\sqrt{2^2+4^2}$
$=\sqrt{4+16}$
$=\sqrt{20}$
$=4.47\ (3\text{ s.f.})$

e $|3\mathbf{i}-5\mathbf{j}|=\sqrt{3^2+5^2}$
$=\sqrt{9+25}$
$=\sqrt{34}$
$=5.83\ (3\text{ s.f.})$

f $|4\mathbf{i}+7\mathbf{j}|=\sqrt{4^2+7^2}$
$=\sqrt{16+49}$
$=\sqrt{65}$
$=8.06\ (3\text{ s.f.})$

g $|-3\mathbf{i}+5\mathbf{j}|=\sqrt{3^2+5^2}$
$=\sqrt{9+25}$
$=\sqrt{34}$
$=5.83\ (3\text{ s.f.})$

h $|-4\mathbf{i}+-\mathbf{j}|=\sqrt{4^2+1^2}$
$=\sqrt{16+1}$
$=\sqrt{17}$
$=4.12\ (3\text{ s.f.})$

2 a $\mathbf{a}=\begin{pmatrix}2\\3\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}3\\-4\end{pmatrix}$ and $\mathbf{c}=\begin{pmatrix}5\\-1\end{pmatrix}$

$\mathbf{a}+\mathbf{b}=\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}3\\-4\end{pmatrix}$
$=\begin{pmatrix}5\\-1\end{pmatrix}$

$|\mathbf{a}+\mathbf{b}|=\sqrt{5^2+(-1)^2}$
$=\sqrt{26}$

b $2\mathbf{a}-\mathbf{c}=2\begin{pmatrix}2\\3\end{pmatrix}-\begin{pmatrix}5\\-1\end{pmatrix}$
$=\begin{pmatrix}-1\\7\end{pmatrix}$

$|2\mathbf{a}-\mathbf{c}|=\sqrt{(-1)^2+7^2}$
$=\sqrt{50}$
$=5\sqrt{2}$

c $3\mathbf{b}-2\mathbf{c}=3\begin{pmatrix}3\\-4\end{pmatrix}-2\begin{pmatrix}5\\-1\end{pmatrix}$
$=\begin{pmatrix}-1\\-10\end{pmatrix}$

$|3\mathbf{b}-2\mathbf{c}|=\sqrt{(-1)^2+(-10)^2}$
$=\sqrt{101}$

3 a a unit vector is $\dfrac{\mathbf{a}}{|\mathbf{a}|}$

$\mathbf{a}=\begin{pmatrix}4\\3\end{pmatrix}$

$|\mathbf{a}|=\sqrt{4^2+3^2}$
$=\sqrt{25}$
$=5$

$\dfrac{\mathbf{a}}{|\mathbf{a}|}=\dfrac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}0.8\\0.6\end{pmatrix}$

3 b a unit vector is $\frac{\mathbf{b}}{|\mathbf{b}|}$

$\mathbf{b} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$|\mathbf{b}| = \sqrt{5^2 + (-12)^2}$

$= \sqrt{169}$

$= 13$

$\frac{\mathbf{b}}{|\mathbf{b}|} = \frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$= \begin{pmatrix} \frac{5}{13} \\ -\frac{12}{13} \end{pmatrix}$

c a unit vector is $\frac{\mathbf{c}}{|\mathbf{c}|}$

$\mathbf{c} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$

$|\mathbf{c}| = \sqrt{(-7)^2 + 24^2}$

$= \sqrt{625}$

$= 25$

$\frac{\mathbf{c}}{|\mathbf{c}|} = \frac{1}{25}\begin{pmatrix} -7 \\ 24 \end{pmatrix}$

$= \begin{pmatrix} -0.28 \\ 0.96 \end{pmatrix}$

d a unit vector is $\frac{\mathbf{d}}{|\mathbf{d}|}$

$\mathbf{d} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

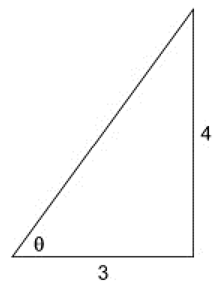
$|\mathbf{d}| = \sqrt{1^2 + (-3)^2}$

$= \sqrt{10}$

$\frac{\mathbf{d}}{|\mathbf{d}|} = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

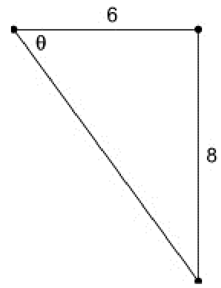
$= \begin{pmatrix} \frac{\sqrt{10}}{10} \\ -\frac{3\sqrt{10}}{10} \end{pmatrix}$

4 a $3\mathbf{i} + 4\mathbf{j}$



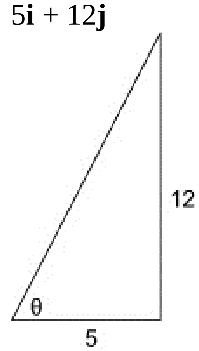
$\tan^{-1}\left(\frac{4}{3}\right) = 53.1°$ above (3 s.f.)

b $6\mathbf{i} - 8\mathbf{j}$



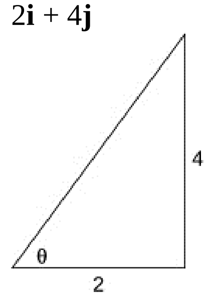
$\tan^{-1}\left(\frac{8}{6}\right) = 53.1°$ below (3 s.f.)

c $5\mathbf{i} + 12\mathbf{j}$



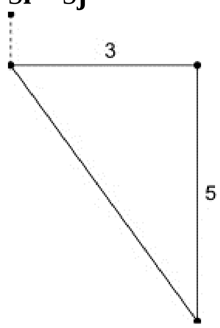
$\tan^{-1}\left(\frac{12}{5}\right) = 67.4°$ above (3 s.f.)

d $2\mathbf{i} + 4\mathbf{j}$



$\tan^{-1}\left(\frac{4}{2}\right) = 63.4°$ above (3 s.f.)

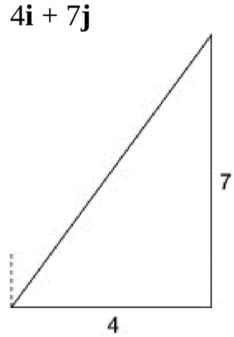
5 a $3\mathbf{i} - 5\mathbf{j}$



$90° + \tan^{-1}\left(\frac{5}{3}\right) = 90° + 59°$

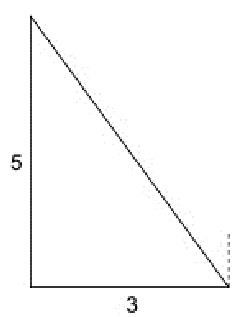
$= 149° \text{ (3 s.f.) to the right}$

b $4\mathbf{i} + 7\mathbf{j}$



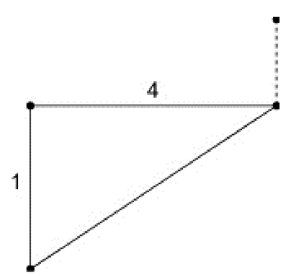
$\tan^{-1}\left(\frac{4}{7}\right) = 29.7° \text{ (3 s.f.) to the right}$

c $-3\mathbf{i} + 5\mathbf{j}$



$\tan^{-1}\left(\frac{3}{5}\right) = 31.0° \text{ (3 s.f.) to the left}$

d $-4\mathbf{i} - \mathbf{j}$



5 d $90° + \tan^{-1}\left(\frac{1}{4}\right) = 90° + 14°$

$= 104° \text{ (3 s.f.) to the left}$

6 a $\cos 45° = \frac{x}{15}$

$x = 15\cos 45°$

$= \frac{15\sqrt{2}}{2}$

$\sin 45° = \frac{y}{15}$

$y = 15\sin 45°$

$= \frac{15\sqrt{2}}{2}$

The vector is $\frac{15\sqrt{2}}{2}\mathbf{i} + \frac{15\sqrt{2}}{2}\mathbf{j}$

or $\begin{pmatrix} \frac{15\sqrt{2}}{2} \\ \frac{15\sqrt{2}}{2} \end{pmatrix}$

b $\cos 20° = \frac{x}{8}$

$x = 8\cos 20°$

$= 7.52$

$\sin 20° = \frac{y}{8}$

$y = 8 \sin 20°$

$= 2.74$

The vector is $7.52\mathbf{i} + 2.74\mathbf{j}$

or $\begin{pmatrix} 7.52 \\ 2.74 \end{pmatrix}$

c $\cos 25° = \frac{x}{20}$

$x = 20 \cos 25°$

$= 18.1$

$\sin 25° = \frac{y}{20}$

$y = 20 \sin 25°$

$= 8.45$

The vector is $18.1\mathbf{i} - 8.45\mathbf{j}$

or $\begin{pmatrix} 18.1 \\ -8.45 \end{pmatrix}$

6 d $\cos 30° = \frac{x}{5}$

$x = 5\cos 30°$

$= \frac{5\sqrt{3}}{2}$

$\sin 30° = \frac{y}{5}$

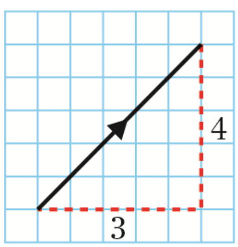
$y = 5\sin 30°$

$= 2.5$

The vector is $\frac{5\sqrt{3}}{2}\mathbf{i} - 2.5\mathbf{j}$

or $\begin{pmatrix} \frac{5\sqrt{3}}{2} \\ -2.5 \end{pmatrix}$

7 a



magnitude $= \sqrt{3^2 + 4^2}$

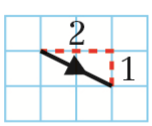
$= \sqrt{25} = 5$

$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

$= 53.1°$ above the positive x-axis

b



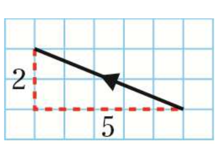
magnitude $= \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26.6°$ below the positive x-axis

c



magnitude $= \sqrt{(-5)^2 + 2^2} = \sqrt{29}$

$\tan\theta = \frac{2}{5}$

7 c $\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$= 21.8°$ above the negative x-axis

$= 158.2°$ above the positive x-axis

8 $|2\mathbf{i} - k\mathbf{j}| = \sqrt{2^2 + (-k)^2} = \sqrt{4 + k^2}$

$\sqrt{4 + k^2} = 2\sqrt{10} = \sqrt{40}$

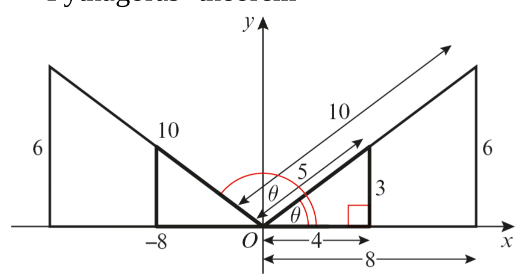
$4 + k^2 = 40$

$k^2 = 36$

$k = \pm 6$

9 $|p\mathbf{i} + q\mathbf{j}| = 10$

Adding the information and using Pythagoras' theorem



$p = 8$ and $q = \pm 6$

Consider also the case where θ is below the x-axis. By symmetry, $p = \pm 8$ and $q = -6$ are also solutions.

So the possible values of p and q are:

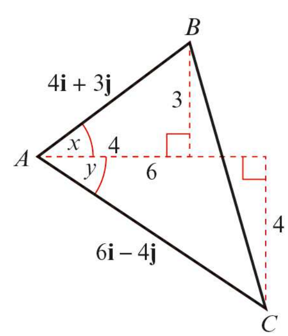
$p = 8, q = 6$

$p = 8, q = -6$

$p = -8, q = 6$

$p = -8, q = -6$

10



a $\tan x = \frac{3}{4}$

$x = \tan^{-1}\frac{3}{4}$

$= 36.870°$

10 b $\tan y = \frac{2}{3}$

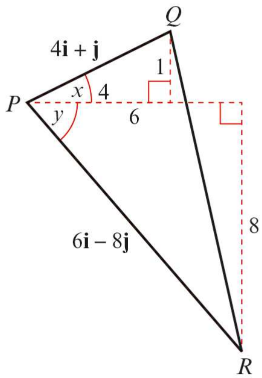
$y = \tan^{-1} \frac{2}{3}$

$= 33.690°$

c Angle $BAC = x + y$

$= 70.6°$ (1 d.p.)

11



a Angle $QPR = x + y$

$\tan x = \frac{1}{4}$

$x = \tan^{-1} \frac{1}{4}$

$= 14.0362...$

$\tan y = \frac{4}{3}$

$y = \tan^{-1} \frac{4}{3}$

$= 53.1301...$

Angle $QPR = 67.2°$ (1 d.p.)

b Area $= \frac{1}{2} rq \sin P$

$r = \sqrt{4^2 + 1^2}$

$= \sqrt{17}$

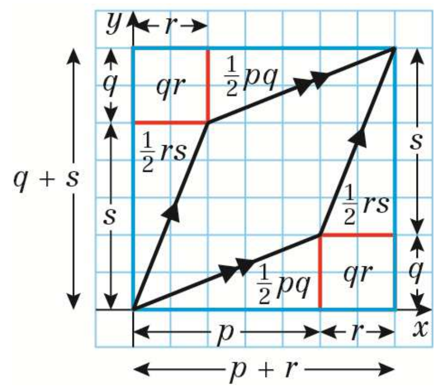
$q = \sqrt{6^2 + 8^2}$

$= \sqrt{100} = 10$

Area $= \frac{1}{2} \times \sqrt{17} \times 10 \times \sin 67.2°$

$= 19.0$ units^2 (3 s.f.)

Challenge



Area of parallelogram

= area of large blue rectangle − 2(area of small red rectangle) − 2(area of triangle 1) − 2(area of triangle 2)

$= (p + r)(q + s) - 2(qr) - 2(\frac{1}{2}pq) - 2(\frac{1}{2}rs)$

$= pq + ps + qr + rs - 2qr - pq - rs$

$= ps - qr$

