Trigonometric identities and equations 10D

1 a Consider $\tan x = -2$
$x = \tan^{-1}(-2)$
$= 63.4°$ (3 s.f.) in the first quadrant
The principal solution marked by A in the diagram is $180° - 63.4° = 116.6°$

b The solutions between 0° and 360°:
$-63.4° + 180° = 116.6°$
$-63.4° + 360° = 296.6°$

2 a $\cos x = 0.4$
$x = \cos^{-1}(0.4)$
$= 66.4$ (3 s.f.)

b $360° - 66.4° = 293.6°$
$180° - 66.4° = 113.6°$
$180° + 66.4° = 246.4°$
$x = 66.4°, 113.6°, 246.4°$ and $293.6°$

3 a Using the graph of $y = \sin\theta$
$\sin\theta = -1$ when $\theta = 270°$

b $\tan\theta = \sqrt{3}$
The calculator solution is 60° $\left(\tan^{-1}\sqrt{3}\right)$
and, as $\tan\theta$ is +ve, θ lies in the first and third quadrants.
$\theta = 60°$ and $(180° + 60°) = 60°, 240°$

c $\cos\theta = \frac{1}{2}$
The calculator solution is 60° and as $\cos\theta$ is +ve, θ lies in the first and fourth quadrants.
$\theta = 60°$ and $(360° - 60°) = 60°, 300°$

d $\sin\theta = \sin 15°$
The acute angle satisfying the equation is $\theta = 15°$.
As $\sin\theta$ is +ve, θ lies in the 1st and 2nd quadrants, so
$\theta = 15°$ and $(180° - 15°) = 15°, 165°$

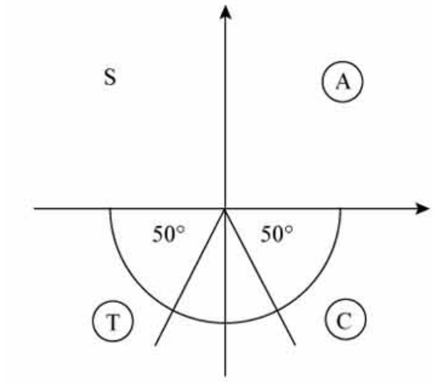
e A first solution is $\cos^{-1}(-\cos 40°) = 140°$
A second solution of $\cos\theta = k$ is $360° -$ 1st solution.
So second solution is 220°.
(Use the quadrant diagram as a check.)

f A first solution is $\tan^{-1}(-1) = -45°$
Use the quadrant diagram, noting that as tan is $-$ve, solutions are in the 2nd and 4th quadrants.
($-45°$ is not in the given interval.)
So solutions are 135° and 315°.

g From the graph of $y = \cos\theta$
$\cos\theta = 0$ when $\theta = 90°, 270°$

h $\sin\theta = -0.766$
$\sin^{-1}(-0.766) = -50°$

$360° - 50° = 310°$

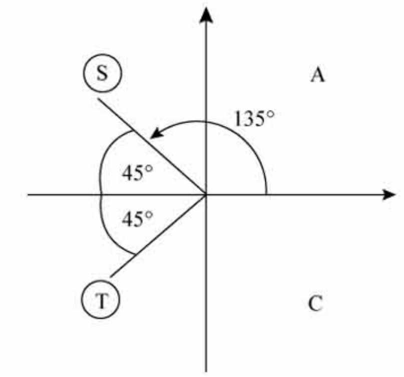


From the diagram, the second solution is $180° + 50° = 230°$.
$\theta = 230°, 310°$

4 a $\sin\theta = \frac{5}{7}$
The first solution is
$\sin^{-1}\left(\frac{5}{7}\right) = 45.6°$
The second solution is
$180° - 45.6° = 134.4°$

b $\cos\theta = -\frac{\sqrt{2}}{2}$
Calculator solution is 135°.
As $\cos\theta$ is $-$ve, θ is in the second and third quadrants.

4 b



Solutions are 135° and 225°

$(135° \text{ and } 360° - 135°)$.

c Calculator solution is

$\cos^{-1}\left(-\frac{2}{3}\right) = 131.8° \text{ (1 d.p.)}$

Second solution is $360° - 131.8° = 228.2°$

d $\sin\theta = -\frac{3}{4}$

$\theta = -48.6°$

$\theta = 360° - 48.6°, \text{or } 180° + 48.6°$

$= 311.4°, 228.6°$

e $\tan\theta = \frac{1}{7}$

$\theta = 8.13° \text{ or } 188°$

f $\tan\theta = \frac{15}{8}$

$\theta = 61.9° \text{ or } 242°$

g $\tan\theta = -\frac{11}{3}$

$\theta = -74.7°$

$\theta = 105.3° \text{ or } 285°$

h $\cos\theta = \frac{\sqrt{5}}{3}$

$\theta = 41.8°, 318°$

5 a $\sqrt{3}\sin\theta = \cos\theta$

So dividing both sides by $\sqrt{3}\cos\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

5 a Calculator solution is 30°.

As $\tan\theta$ is + ve, θ is in the first and third quadrants.

Solutions are 30°, 210°

$(30° \text{ and } 180° + 30°)$.

b $\sin\theta + \cos\theta = 0$

So $\sin\theta = -\cos\theta \Rightarrow \tan\theta = -1$

Calculator solution $(-45°)$ is not in the given interval.

As $\tan\theta$ is − ve, θ is in the second and fourth quadrants.

Solutions are 135° and 315°

$(180° + \tan^{-1}(-1), 360° + \tan^{-1}(-1))$.

c $3\sin\theta = 4\cos\theta$

$\tan\theta = \frac{4}{3}$

$\theta = 53.1° \text{ or } 233°$

d $2\sin\theta - 3\cos\theta = 0$

$\tan\theta = \frac{3}{2}$

$\theta = 56.3° \text{ or } 236°$

e $\sqrt{2}\sin\theta = 2\cos\theta$

$\tan\theta = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\theta = 54.7° \text{ or } 235°$

f $\sqrt{5}\sin\theta + \sqrt{2}\cos\theta = 0$

$\sqrt{5}\tan\theta + \sqrt{2} = 0$

$\tan\theta = -\frac{\sqrt{2}}{\sqrt{5}}$

$\theta = -32.3° \quad \theta > 0$

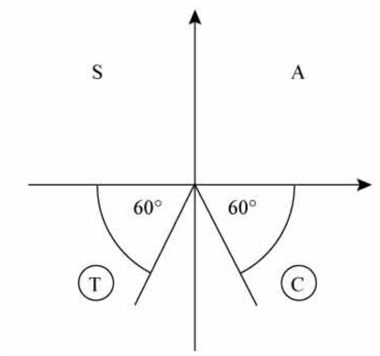
$\theta = 148° \text{ or } 328°$

6 a Calculator solution of

$\sin x° = -\frac{\sqrt{3}}{2}$ is $x = -60°$

As $\sin x°$ is − ve, x is in the third and fourth quadrants.

6 a



Read off all solutions in the interval $-180° \leq x \leq 540°$.

$x = -120°, -60°, 240°, 300°$

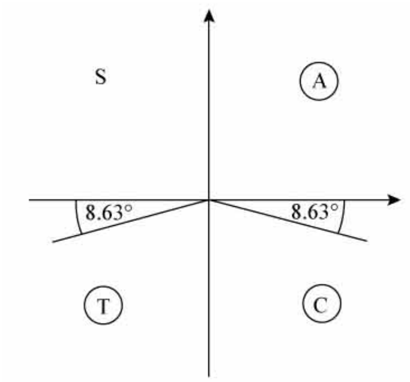
b $2\sin x° = -0.3$

$\sin x° = -0.15$

First solution is $x = \sin^{-1}(-0.15)$

$= -8.63°\ (3\text{ s.f.})$

As $\sin x°$ is $-$ve, x is in the third and fourth quadrants.



Read off all solutions in the interval $-180° \leq x \leq 180°$.

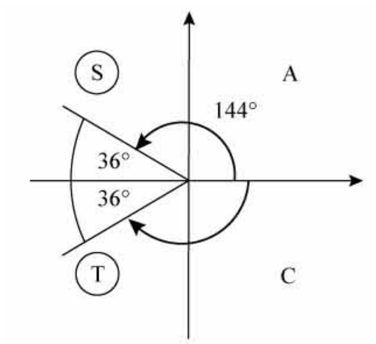
$x = -171.37°,\ -8.63°$

$x = -171°,\ -8.63°\ (3\text{ s.f.})$

c $\cos x° = -0.809$

Calculator solution is $144°\ (3\text{ s.f.})$

As $\cos x°$ is $-$ve, x is in the second and third quadrants.



c Read off all the solutions in the interval $-180° \leq x \leq 180°$.

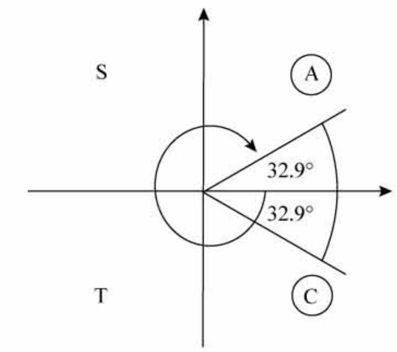
$x = -144°, +144°$

Note: Here solutions are $\cos^{-1}(-0.809)$ and $(360° - \cos^{-1}(-0.809))$.

d $\cos x° = 0.84$

Calculator solution is $32.9°\ (3\text{ s.f.})$ (not in interval).

As $\cos x°$ is $+$ve, x is in the first and fourth quadrants.



Read off all the solutions in the interval $-360° < x < 0°$.

$x = -327°, -32.9°\ (3\text{ s.f.})$

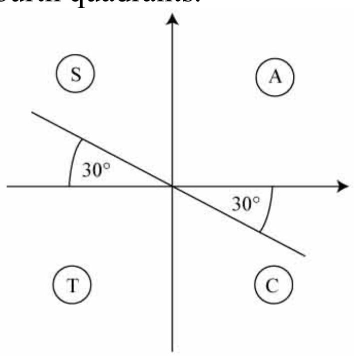
(*Note*: Here solutions are $\cos^{-1}(0.84) - 360°$ and $\left(360° - \cos^{-1}(0.84)\right) - 360°$)

e $\tan x° = -\dfrac{\sqrt{3}}{3}$

Calculator solution is

$\tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right) = -30°$ (not in interval)

As $\tan x°$ is $-$ve, x is in the second and fourth quadrants.



Read off all the solutions in the interval $0° \leq x \leq 720°$.

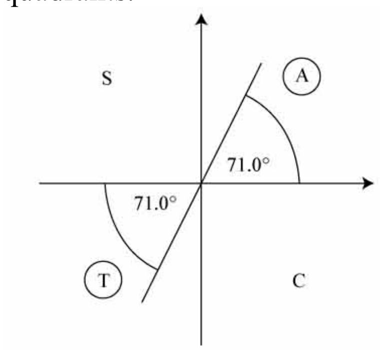
$x = 150°, 330°, 510°, 690°$

6 e *Note*: Here solutions are

$$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)+180°,\ \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)+360°,$$

$$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)+540°, \cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)+720°.$$

f $\tan x° = 2.90$
Calculator solution is
$\tan^{-1}(2.90) = 71.0°(3\text{ s.f.})$
(not in interval).
As $\tan x°$ is +ve, x is in the first and third quadrants.



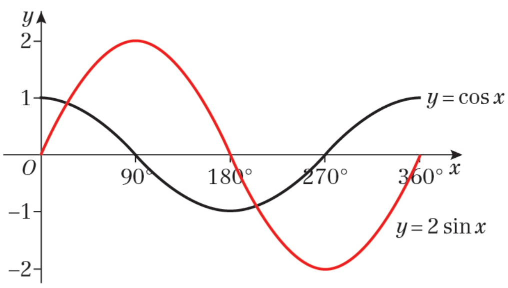
Read off all solutions in the interval
$80° \le x \le 440°$.
$x = 251°, 431°$
(*Note*: Here solutions are
$\tan^{-1}(2.90)+180°,\ \tan^{-1}(2.90)+360°$.)

7 a It should be $\tan x = \frac{2}{3}$, not $\frac{3}{2}$.

b Squaring both sides creates extra solutions.

c $\tan x = \frac{2}{3}$
$x = 33.7°$ or $x = -146.3°$

8 a



b The graphs intersect at 2 points in the given range so there are 2 solutions.

c $2 \sin x = \cos x$
$\frac{\sin x}{\cos x} = \frac{1}{2}$
$\tan x = \frac{1}{2}$
$x = 26.6°$
$x = 26.6° + 180° = 206.6°$
$x = 26.6°$ or $206.6°$

9 $\tan \theta = \pm 3$
When $\tan \theta = 3$, $\theta = 71.6°$
or $\theta = 71.6° + 180° = 251.6°$
When $\tan \theta = -3$, $\theta = -71.6°$
or $\theta = -71.6° + 180° = 108.4°$ or
$\theta = 108.4° + 180° = 288.4°$
$\theta = 71.6°, 108.4°, 251.6°$ or $288.4°$

10 a $4 \sin^2 x - 3 \cos^2 x = 2$
$4 \sin^2 x - 3(1 - \sin^2 x) = 2$
$4 \sin^2 x - 3 + 3\sin^2 x = 2$
$7 \sin^2 x = 5$

b $\sin^2 x = \frac{5}{7}$
$\sin x = \pm\sqrt{\frac{5}{7}}$
$x = 57.7°$ or $-57.7°$
$x = 180° - 57.7° = 122.3°$
$x = 180° + 57.7° = 237.7°$
$x = 360° - 57.7° = 302.3°$
$x = 57.7°, 122.3°, 237.7°$ or $302.3°$

11 a $2 \sin^2 x + 5 \cos^2 x = 1$
$2 \sin^2 x + 5(1 - \sin^2 x) = 1$
$2 \sin^2 x + 5 - 5 \sin^2 x = 1$
$3 \sin^2 x = 4$

b Using $3 \sin^2 x = 4$
$\sin^2 x = \frac{4}{3}$
$\sin x > 1$, therefore there are no solutions.

