Trigonometric identities and equations 10B

1 **a** $\sin 135° = +\sin 45°$
(135° is in the second quadrant
at 45° to the horizontal.)
So $\sin 135° = \frac{\sqrt{2}}{2}$

b $\sin(-60)° = -\sin 60°$
(–60° is in the fourth quadrant
at 60° to the horizontal.)
So $\sin(-60°) = -\frac{\sqrt{3}}{2}$

c $\sin 330° = -\sin 30°$
(330° is in the fourth quadrant
at 30° to the horizontal.)
So $\sin 330° = -\frac{1}{2}$

d $\sin 420° = +\sin 60°$
(on second revolution)
So $\sin 420° = \frac{\sqrt{3}}{2}$

e $\sin(-300°) = +\sin 60°$
(–300° is in the first quadrant
at 60° to the horizontal.)
So $\sin(-300°) = \frac{\sqrt{3}}{2}$

f $\cos 120° = -\cos 60°$
(120° is in the second quadrant
at 60° to the horizontal.)
So $\cos 120° = -\frac{1}{2}$

g $\cos 300° = +\cos 60°$
(300° is in the fourth quadrant
at 60° to the horizontal.)
So $\cos 300° = \frac{1}{2}$

h $\cos 225° = -\cos 45°$
(225° is in the third quadrant
at 45° to the horizontal.)
So $\cos 225° = -\frac{\sqrt{2}}{2}$

i $\cos(-210°) = -\cos 30°$
(–210° is in the second quadrant
at 30° to the horizontal.)
So $\cos(-210°) = -\frac{\sqrt{3}}{2}$

j $\cos 495° = -\cos 45°$
(495° is in the second quadrant
at 45° to the horizontal.)
So $\cos 495° = -\frac{\sqrt{2}}{2}$

k $\tan 135° = -\tan 45°$
(135° is in the second quadrant
at 45° to the horizontal.)
So $\tan 135° = -1$

l $\tan(-225°) = -\tan 45°$
(–225° is in the second quadrant
at 45° to the horizontal.)
So $\tan(-225°) = -1$

m $\tan 210° = +\tan 30°$
(210° is in the third quadrant
at 30° to the horizontal.)
So $\tan 210° = \frac{\sqrt{3}}{3}$

n $\tan 300° = -\tan 60°$
(300° is in the fourth quadrant
at 60° to the horizontal.)
So $\tan 300° = -\sqrt{3}$

o $\tan(-120°) = +\tan 60°$
(–120° is in the third quadrant
at 60° to the horizontal.)
So $\tan(-120°) = \sqrt{3}$

Challenge

a i $\tan 30° = \frac{1}{CE}$

$CE = \frac{1}{\tan 30°}$

$= \frac{1}{\frac{\sqrt{3}}{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{3}$

$= \sqrt{3}$

ii Using Pythagoras' theorem

$CD^2 = 1^2 + \sqrt{3}^2$

$CD = \sqrt{1+3}$

$CD = 2$

iii Using Pythagoras' theorem on the isosceles triangle ABC

$AB^2 + BC^2 = (1+\sqrt{3})^2$

$AB = BC$ so $BC^2 + BC^2 = (1+\sqrt{3})^2$

$2BC^2 = 4 + 2\sqrt{3}$

$BC^2 = 2 + \sqrt{3}$

$BC = \sqrt{2+\sqrt{3}}$

iv $DB = AB - AD$

Using Pythagoras' theorem

$AD = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

$DB = \sqrt{2+\sqrt{3}} - \sqrt{2}$

b Angle $BCD = 45° - 30° = 15°$

c i $\sin 15° = \frac{DB}{CD}$

$= \frac{\sqrt{2+\sqrt{3}} - \sqrt{2}}{2}$

ii $\cos 15° = \frac{BC}{CD}$

$= \frac{\sqrt{2+\sqrt{3}}}{2}$

