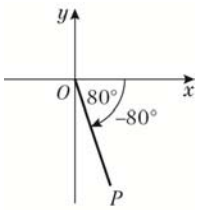
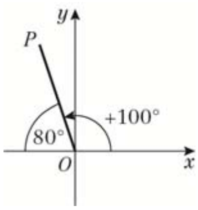
Trigonometric identities and equations 10A

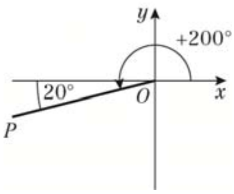
1 a



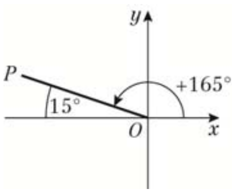
b



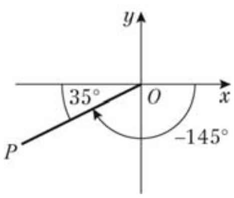
c



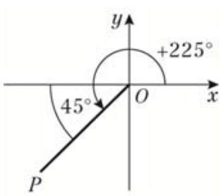
d



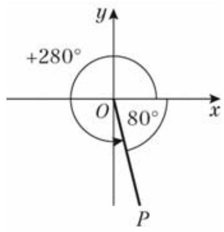
e



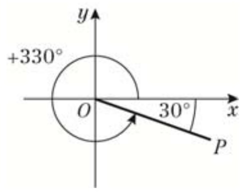
f



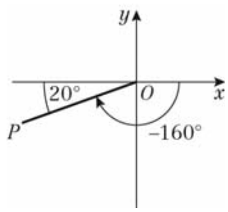
g



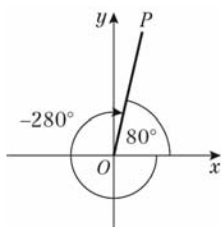
h



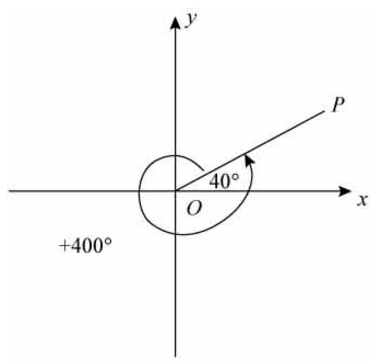
i



j

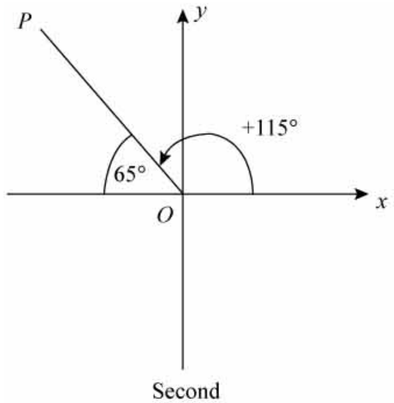


2 a

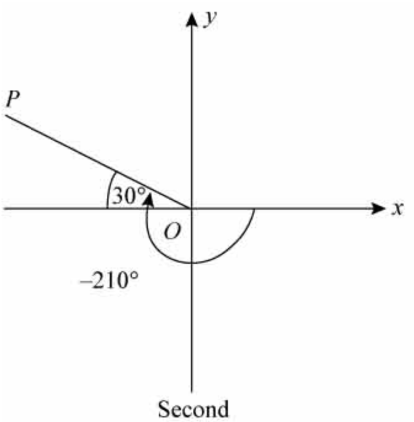


First

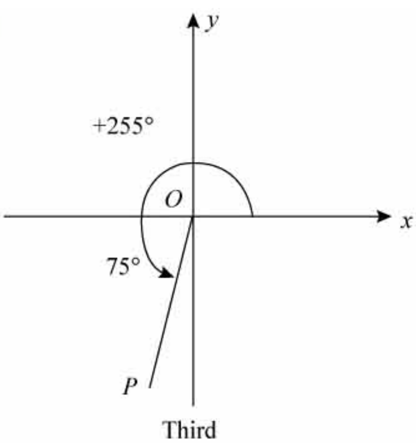
2 b



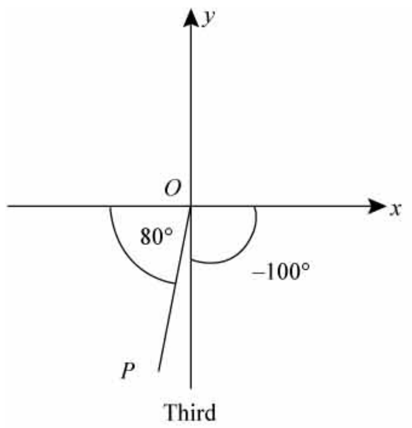
c



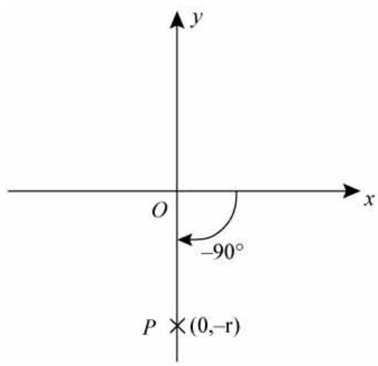
d



2 e

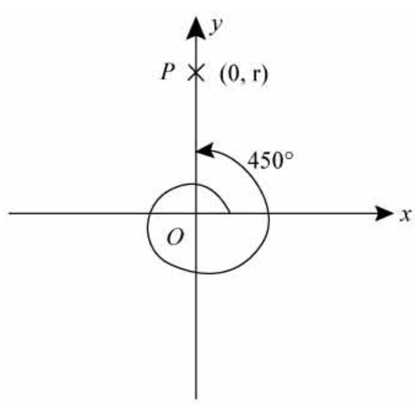


3 a



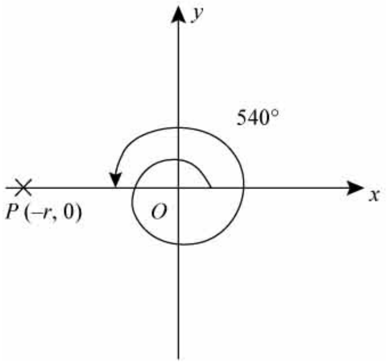
$$\sin(-90)^\circ = \frac{-r}{r} = -1$$

b



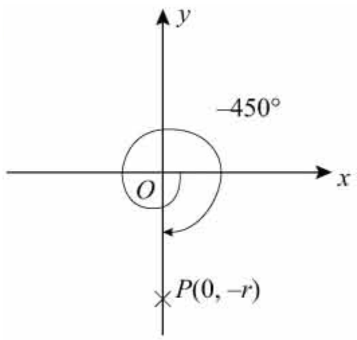
$$\sin 450^\circ = \frac{r}{r} = 1$$

3 c



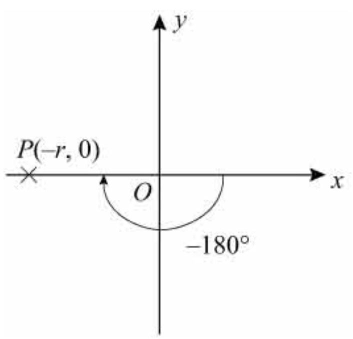
$$\sin 540° = \frac{0}{r} = 0$$

d



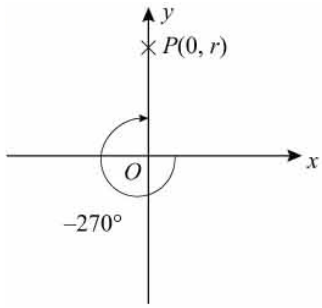
$$\sin(-450)° = \frac{-r}{r} = -1$$

e



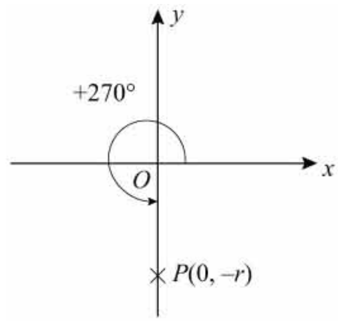
$$\cos(-180)° = \frac{-r}{r} = -1$$

f



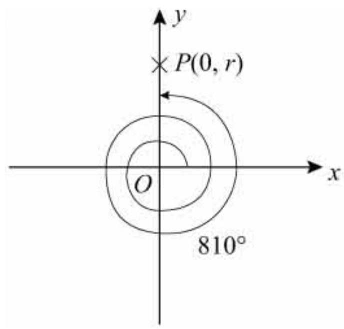
$$\cos(-270)° = \frac{0}{r} = 0$$

g



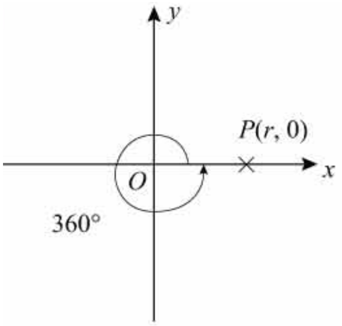
$$\cos 270° = \frac{0}{r} = 0$$

h



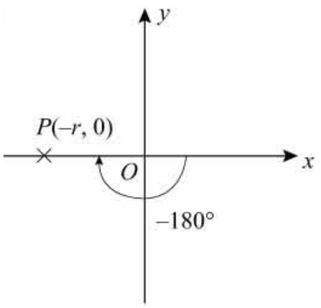
$$\cos 810° = \frac{0}{r} = 0$$

3 i



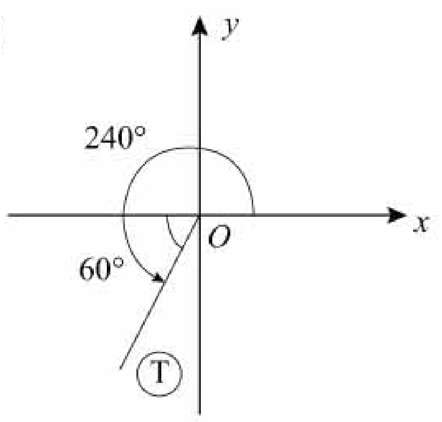
$\tan 360° = \frac{0}{r} = 0$

j



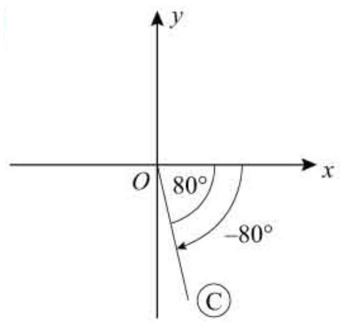
$\tan(-180)° = \frac{0}{-r} = 0$

4 a



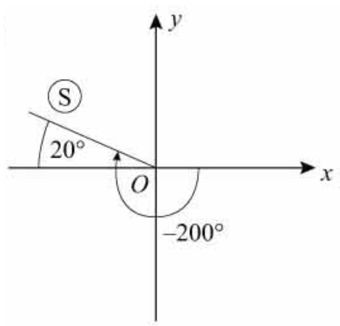
60° is the acute angle.
In the third quadrant sin is −ve.
So $\sin 240° = -\sin 60°$

4 b



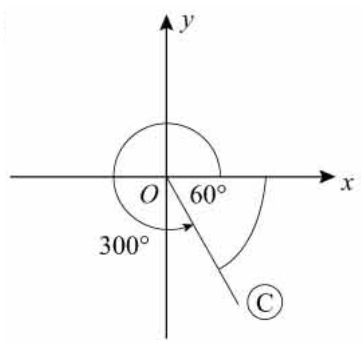
80° is the acute angle.
In the fourth quadrant sin is −ve.
So $\sin(-80)° = -\sin 80°$

c



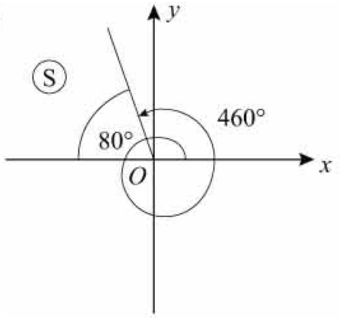
20° is the acute angle.
In the second quadrant sin is +ve.
So $\sin(-200)° = +\sin 20°$

d



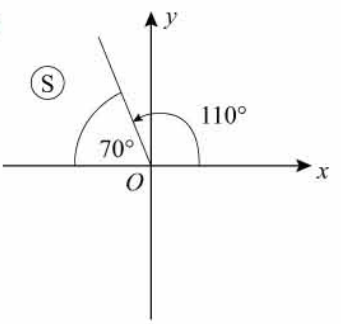
60° is the acute angle.
In the fourth quadrant sin is −ve.
So $\sin 300° = -\sin 60°$

4 e



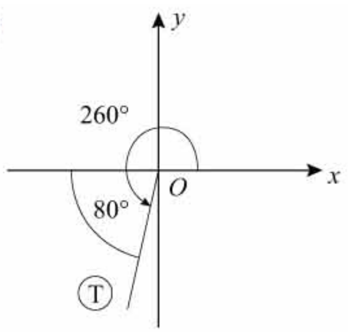
80° is the acute angle.
In the second quadrant sin is + ve.
So $\sin 460° = +\sin 80°$

f



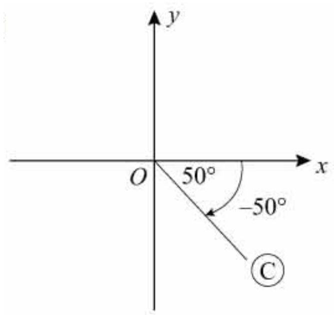
70° is the acute angle.
In the second quadrant cos is − ve.
So $\cos 110° = -\cos 70°$

g



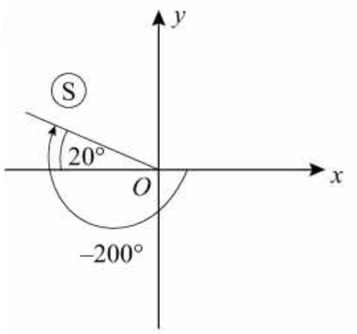
80° is the acute angle.
In the third quadrant cos is − ve.
So $\cos 260° = -\cos 80°$

h



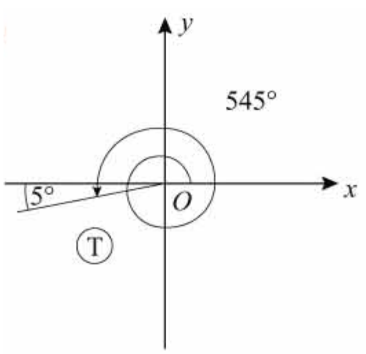
50° is the acute angle.
In the fourth quadrant cos is + ve.
So $\cos(-50)° = +\cos 50°$

i



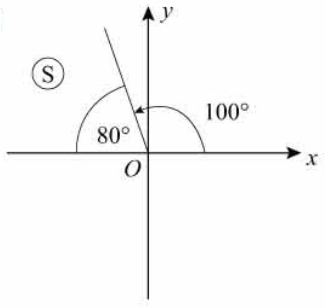
20° is the acute angle.
In the second quadrant cos is − ve.
So $\cos(-200)° = -\cos 20°$

j



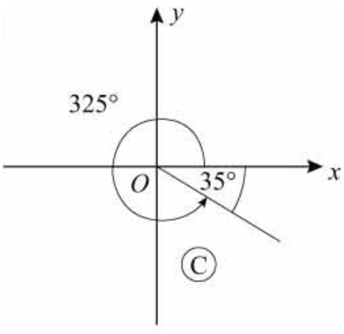
5° is the acute angle.
In the third quadrant cos is − ve.
So $\cos 545° = -\cos 5°$

4 k



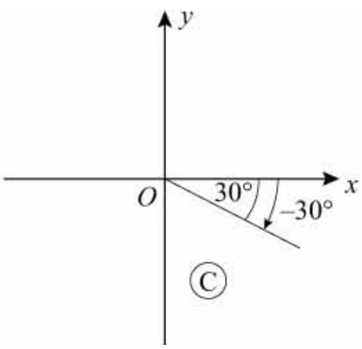
80° is the acute angle.
In the second quadrant tan is $-$ve.
So $\tan 100° = -\tan 80°$

l



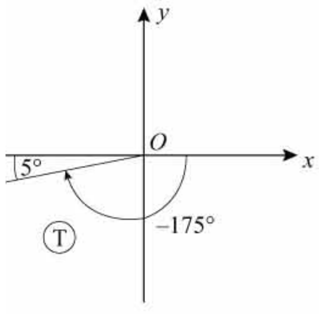
35° is the acute angle.
In the fourth quadrant tan is $-$ve.
So $\tan 325° = -\tan 35°$

m



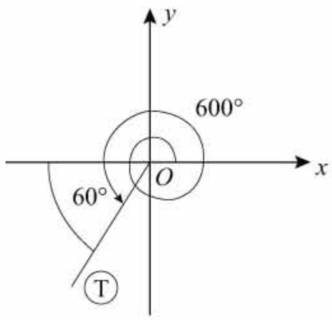
30° is the acute angle.
In the fourth quadrant tan is $-$ve.
So $\tan(-30)° = -\tan 30°$

n



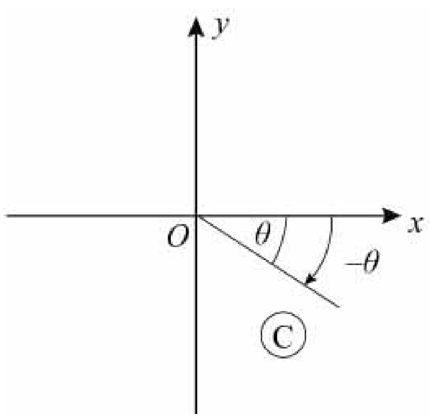
5° is the acute angle.
In the third quadrant tan is $+$ve.
So $\tan(-175)° = +\tan 5°$

o



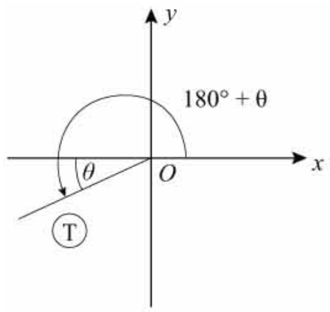
60° is the acute angle.
In the third quadrant tan is $+$ve.
So $\tan 600° = +\tan 60°$

5 a



sin is $-$ve in this quadrant.
So $\sin(-\theta) = -\sin\theta$

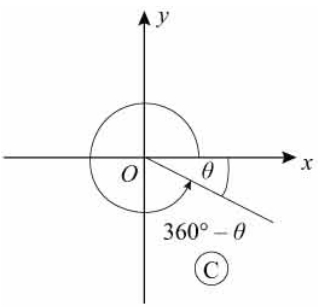
5 b



sin is $-$ve in this quadrant.

So $\sin(180°+\theta) = -\sin\theta$

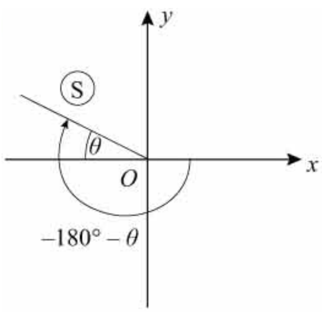
c



sin is $-$ve in this quadrant.

So $\sin(360°-\theta) = -\sin\theta$

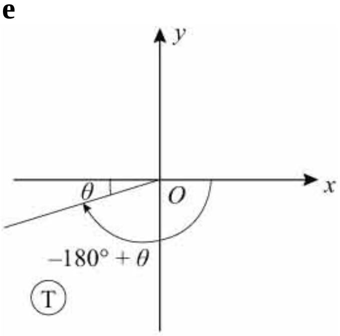
d



sin is $+$ve in this quadrant.

So $\sin-(180°+\theta) = +\sin\theta$

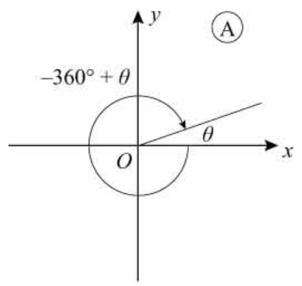
e



sin is $-$ve in this quadrant.

So $\sin(-180°+\theta) = -\sin\theta$

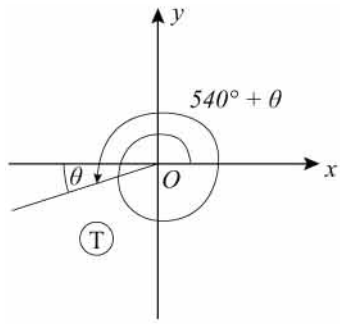
f



sin is $+$ve in this quadrant.

So $\sin(-360°+\theta) = +\sin\theta$

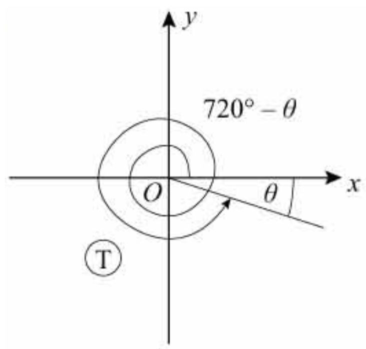
g



sin is $-$ve in this quadrant.

So $\sin(540°+\theta) = -\sin\theta$

h



sin is $+$ve in this quadrant.

So $\sin(720°-\theta) = -\sin\theta$

i $\theta + 720°$ is in the first quadrant with θ to the horizontal.

So $\sin(\theta + 720°) = +\sin\theta$

6 a $180° - \theta$ is in the second quadrant where cos is –ve, and the angle to the horizontal is θ.
So $\cos(180° - \theta) = -\cos\theta$

6 b $180° + \theta$ is in the third quadrant, at θ to the horizontal.
So $\cos(180° + \theta) = -\cos\theta$

c $-\theta$ is in the fourth quadrant, at θ to the horizontal.
So $\cos(-\theta) = +\cos\theta$

d $-(180° - \theta)$ is in the third quadrant, at θ to the horizontal.
So $\cos -(180° - \theta) = -\cos\theta$

e $\theta - 360°$ is in the first quadrant, at θ to the horizontal.
So $\cos(\theta - 360°) = \cos\theta$

f $\theta - 540°$ is in the third quadrant, at θ to the horizontal.
So $\cos(\theta - 540°) = -\cos\theta$

g $-\theta$ is in the fourth quadrant.
So $\tan(-\theta) = -\tan\theta$

h $(180° - \theta)$ is in the second quadrant.
So $\tan(180° - \theta) = -\tan\theta$

i $(180° + \theta)$ is in the third quadrant.
So $\tan(180° + \theta) = +\tan\theta$

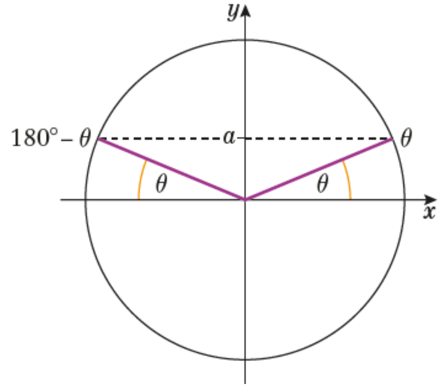
j $(-180° + \theta)$ is in the third quadrant.
So $\tan(-180° + \theta) = +\tan\theta$

k $(540° - \theta)$ is in the second quadrant.
So $\tan(540° - \theta) = -\tan\theta$

l $(\theta - 360°)$ is in the first quadrant.
So $\tan(\theta - 360°) = +\tan\theta$

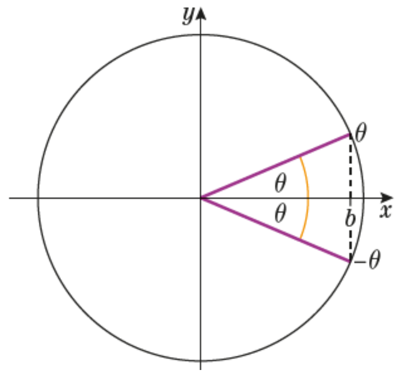
Challenge

a Diagram showing the positions of θ and $(180° - \theta)$ on the unit circle:



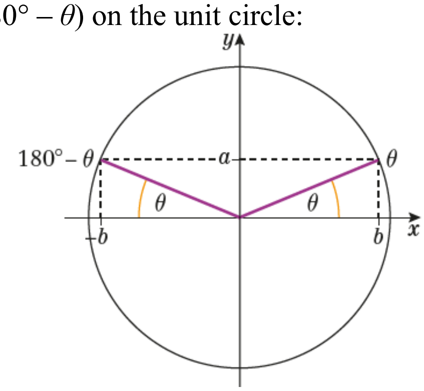
From the diagram, $\sin\theta = a$
and $\sin(180° - \theta) = a$
so $\sin\theta = \sin(180° - \theta)$

b Diagram showing the positions of $-\theta$ and θ on the unit circle:



From the diagram, $\cos\theta = b$
and $\cos(-\theta) = b$
so $\cos\theta = \cos(-\theta)$

c Diagram showing the positions of θ and $(180° - \theta)$ on the unit circle:



From the diagram, $\tan\theta = \dfrac{a}{b}$

and $\tan(180° - \theta) = -\dfrac{a}{b}$

so $\tan(180° - \theta) = -\tan(\theta)$

