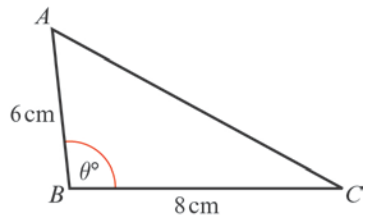
Trigonometric ratios, Mixed Exercise 9

1



a Using area of $\Delta ABC = \frac{1}{2}ac\sin B$

$10\text{ cm}^2 = \frac{1}{2}\times 6\times 8\times \sin\theta°\text{cm}^2$

So $10 = 24\sin\theta°$

So $\sin\theta° = \frac{10}{24} = \frac{5}{12}$

$\Rightarrow \theta = 24.6 \text{ or } 155\ (3\text{ s.f.})$

As θ is obtuse, $\angle ABC = 155°\ (3\text{ s.f.})$

b Using the cosine rule

$b^2 = a^2 + c^2 - 2ac\cos B$

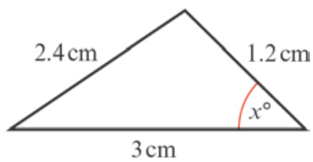
$AC^2 = 8^2 + 6^2 - 2\times 8\times 6\times \cos B$

$= 187.26\ldots$

$AC = 13.68\ldots$

The third side has length 13.7 m (3 s.f.).

2 a



Using the cosine rule

$$\cos x° = \frac{3^2 + 1.2^2 - 2.4^2}{2\times 3\times 1.2}$$

$= 0.65$

$x = \cos^{-1}(0.65)$

$= 49.458\ldots$

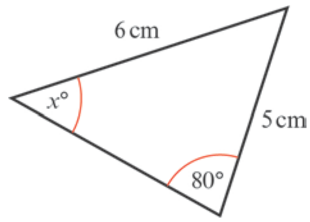
$x = 49.5\ (3\text{ s.f.})$

Using the area of a triangle formula

area $= \frac{1}{2}\times 1.2\times 3\times \sin x°\text{cm}^2$

$= 1.37\text{ cm}^2\ (3\text{ s.f.})$

2 b



Using the sine rule

$$\frac{\sin x°}{5} = \frac{\sin 80°}{6}$$

$$\sin x° = \frac{5\sin 80°}{6}$$

$= 0.8206\ldots$

$x = 55.2\ (3\text{ s.f.})$

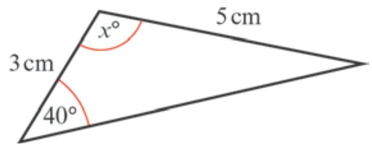
The angle between the 5 cm and 6 cm sides is $180° - (80 + x)° = (100 - x)°$.

Using the area of a triangle formula:

area $= \frac{1}{2}\times 5\times 6\times \sin(100 - x)°\text{ cm}^2$

$= 10.6\text{ cm}^2\ (3\text{ s.f.})$

c



Use the sine rule to find the angle opposite the 3 cm side. Call this $y°$.

$$\frac{\sin y°}{3} = \frac{\sin 40°}{5}$$

$$\sin y° = \frac{3\sin 40°}{5}$$

$\Rightarrow y = 22.68\ldots$

So $x = 180 - (40 + y)$

$= 117\ (3\text{ s.f.})$

Area of triangle $= \frac{1}{2}\times 3\times 5\times \sin x°$

$= 66.6\text{ cm}^2\ (3\text{ s.f.})$

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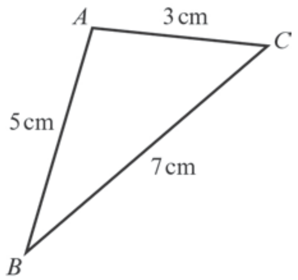
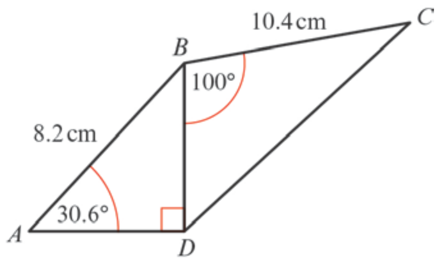


Use the cosine rule to find angle A.

$$\cos A = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5}$$
$$= -0.5$$
$$A = \cos^{-1}(-0.5)$$
$$= 120°$$

$$\text{Area of triangle} = \tfrac{1}{2} \times 3 \times 5 \times \sin A \text{ cm}^2$$
$$= 6.495\ldots \text{cm}^2$$
$$= 6.50 \text{ cm}^2 \text{ (3 s.f.)}$$

4 a



In ΔBDA, $\dfrac{BD}{8.2} = \sin 30.6°$

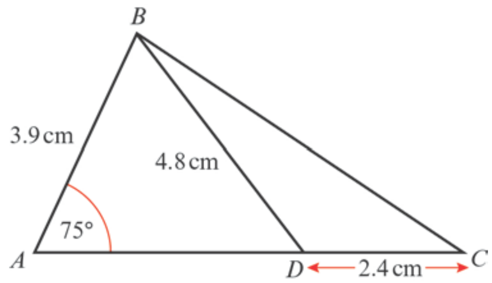
So $BD = 8.2 \sin 30.6°$
$$= 4.174\ldots$$

$$\frac{AD}{8.2} = \cos 30.6$$
$$AD = 8.2\cos 30.6 = 7.0580...$$
$$\angle ABD = 90° - 30.6°$$
$$= 59.4°$$

We can use AD and BD to calculate the area of ΔABD or use:

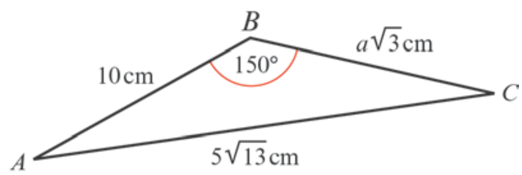
$$\text{Area of } \Delta ABD = \tfrac{1}{2} \times 8.2 \times BD \times \sin 59.4°$$
$$= 14.7307\ldots \text{cm}^2$$
$$\text{Area of } \Delta BDC = \tfrac{1}{2} \times 10.4 \times BD \times \sin 100°$$
$$= 21.375\ldots \text{cm}^2$$
$$\text{Total area} = \text{area } \Delta ABD + \text{area } \Delta BDC$$
$$= 36.1 \text{ cm}^2 \text{ (3 s.f.)}$$

4 b



$$\frac{\sin \angle ADB}{3.9} = \frac{\sin 75°}{4.8}$$
$$\sin \angle ADB = \frac{3.9 \sin 75°}{4.8}$$
$$\angle ADB = \sin^{-1}\left(\frac{3.9 \sin 75°}{4.8}\right)$$
$$= 51.7035\ldots°$$
$$\text{So } \angle ABD = 180° - (75 + \angle ADB)°$$
$$= 53.296\ldots°$$
$$\text{Area of } \Delta ABD = \tfrac{1}{2} \times 3.9 \times 4.8 \times \sin \angle ABD$$
$$= 7.504\ldots \text{cm}^2$$
$$\text{In } \Delta BDC, \angle BDC = 180° - \angle ADB$$
$$= 128.29\ldots°$$
$$\text{Area of } \Delta BDC = \tfrac{1}{2} \times 2.4 \times 4.8 \times \sin \angle BDC$$
$$= 4.520\ldots \text{cm}^2$$
$$\text{Total area} = \text{area } \Delta ABD + \text{area } \Delta BDC$$
$$= 12.0 \text{ cm}^2 \text{ (3 s.f.)}$$

5



a Using the cosine rule:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\left(5\sqrt{13}\right)^2 = \left(a\sqrt{3}\right)^2 + 10^2$$
$$-2 \times a\sqrt{3} \times 10 \times \cos 150°$$
$$325 = 3a^2 + 100 + 30a$$
$$3a^2 + 30a - 225 = 0$$
$$a^2 + 10a - 75 = 0$$
$$(a+15)(a-5) = 0$$
$$\Rightarrow a = 5 \text{ as } a > 0$$

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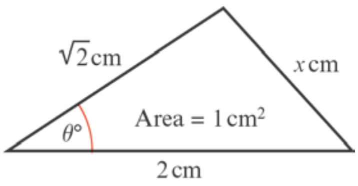
5 b Area $\Delta ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} \times \sin 150°$

$= 12.5\sqrt{3}\ \text{cm}^2$

6



Using the area formula:

$1 = \frac{1}{2} \times 2 \times \sqrt{2} \times \sin\theta°$

$\Rightarrow \sin\theta° = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45 \text{ or } 135$

But as θ is not the largest angle, θ must be 45.

Use the cosine rule to find x.

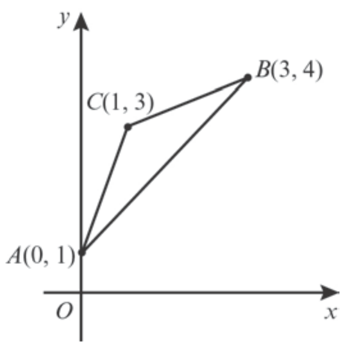
$x^2 = 2^2 + \left(\sqrt{2}\right)^2 - 2 \times 2 \times \sqrt{2} \times \cos 45°$

$x^2 = 4 + 2 - 4 = 2$

So $x = \sqrt{2}$

So the triangle is isosceles with two angles of 45°. It is a right-angled isosceles triangle.

7



a Use Pythagoras' theorem.

$AC = \sqrt{(1-0)^2 + (3-1)^2}$

$= \sqrt{5}$

$= b$

$BC = \sqrt{(3-1)^2 + (4-3)^2}$

$= \sqrt{5}$

$= a$

7 a $AB = \sqrt{(3-0)^2 + (4-1)^2}$

$= \sqrt{18}$

$= c$

Using the cosine rule

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}}$

$= \frac{-8}{10}$

$= \frac{-4}{5}$

Find sin C by using the identity,

$\cos^2 x + \sin^2 x = 1$ or by drawing a 3,4,5 triangle and looking at the ratio of the sides.

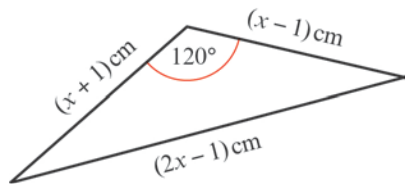
b Using the area formula:

area of $\Delta ABC = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times \sqrt{5} \times \sqrt{5} \times \sin C$

$= 1.5\ \text{cm}^2$

8



a Using the cosine rule

$(2x-1)^2 = (x+1)^2 + (x-1)^2$

$\quad - 2(x+1)(x-1)\cos 120°$

$4x^2 - 4x + 1 = \left(x^2 + 2x + 1\right)$

$\quad + \left(x^2 - 2x + 1\right) + \left(x^2 - 1\right)$

$4x^2 - 4x + 1 = 3x^2 + 1$

$x^2 - 4x = 0$

$x(x-4) = 0$

$\Rightarrow x = 4 \quad x > 1$

b Area of $\Delta = \frac{1}{2} \times (x+1) \times (x-1) \times \sin 120°$

$= \frac{1}{2} \times 5 \times 3 \times \sin 120°$

$= \frac{1}{2} \times 5 \times 3 \times \frac{\sqrt{3}}{2}$

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8 b Area of $\Delta = \dfrac{15\sqrt{3}}{4}$

$= 6.50 \text{ cm}^2$ (3 s.f.)

9 a $b^2 = a^2 + c^2 - 2ac\cos B$

$= 1.4^2 + 1.2^2 - 2 \times 1.4 \times 1.2 \times \cos 70°$

$= 1.96 + 1.44 - 1.149\,187\,68$

So $b = 1.500\,027\ldots$

So point C is 1.50 km from the park keeper's hut.

b $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\dfrac{\sin A}{1.4} = \dfrac{\sin 70°}{1.5}$

$\sin A = \dfrac{1.4 \sin 70°}{1.5}$

So $A = 61.288\,10°$

Bearing $= 360° - (180° - 61.288\,10°)$

$= 241.29°$

The bearing of the hut from point C is 241°.

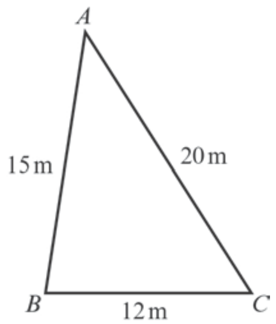
c Area of $\Delta = \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 1.4 \times 1.2 \times \sin 70°$

$= 0.789\,34\ldots$

$= 0.789 \text{ km}^2$ (3 s.f.)

10



$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{20^2 + 15^2 - 12^2}{2(20)(15)}$

$\cos A = \dfrac{400 + 225 - 144}{600}$

So $A = 36.7°$

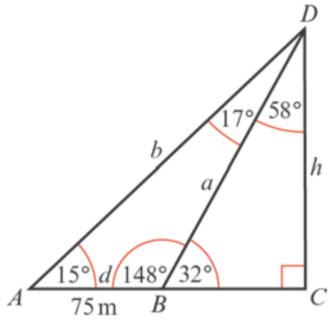
Area of one sail $= \frac{1}{2}bc\sin A$

$= \frac{1}{2} \times 20 \times 15 \times \sin 36.7°$

$= 89.665\ldots$

Area of all four sails $= 359 \text{ m}^2$ (3 s.f.)

11



Using triangle ABD, the angles are 15°, 148° and 17°.

$\dfrac{b}{\sin B} = \dfrac{d}{\sin D}$

$\dfrac{b}{\sin 148°} = \dfrac{75}{\sin 17°}$

$b = \dfrac{75 \sin 148°}{\sin 17°}$

$b = 135.936\ldots$

Using the larger right-angled triangle:

$\sin 15° = \dfrac{\text{height}}{135.936}$

height $= 135.936 \sin 15°$

$= 35.1829\ldots$

The height of the church tower is 35.2 m (3 s.f.).

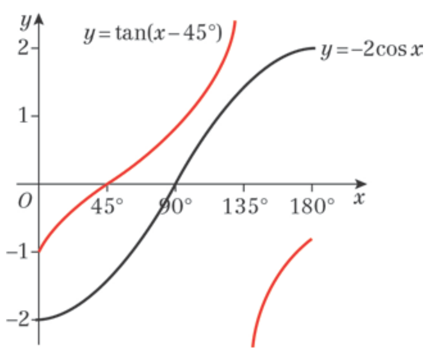
12 a A stretch of scale factor 2 in the x direction.

b A translation of +3 in the y direction.

c A reflection in the x-axis.

d A translation of 20 in the negative x direction (i.e. 20 to the left).

13 a



b $\tan(x - 45°) + 2\cos x = 0$

$\tan(x - 45°) = -2\cos x$

The graphs do not intersect so there are no solutions.

14 a As it is the graph of $y = \sin x°$ translated, the gap between A and B is 180, so $p = 300$.

b The difference in the x-coordinates of D and A is 90, so the x-coordinate of D is 30. The maximum value of y is 1, so D is the point (30, 1).

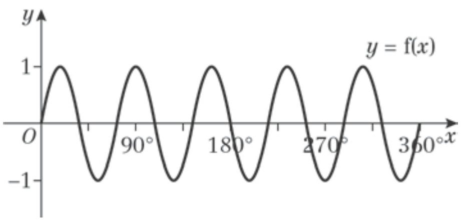
c For the graph of $y = \sin x°$, the first positive intersection with the x-axis would occur at 180. The point A is at 120 and so the curve has been translated by 60 to the left.

$k = 60$

d The equation of the curve is $y = \sin(x + 60)°$.

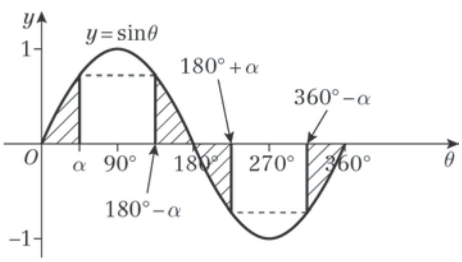
When $x = 0,\ y = \sin 60° = \frac{\sqrt{3}}{2}$, so $q = \frac{\sqrt{3}}{2}$.

15 a The graph of $y = \sin x$ crosses the x-axis at (180°, 0).
f(x) = sin px is a stretch horizontally with scale factor $\frac{36}{180} = \frac{1}{5}$.
f(x) = sin 5x
$p = 5$



b The period of f(x) is 360 ÷ 5 = 72°.

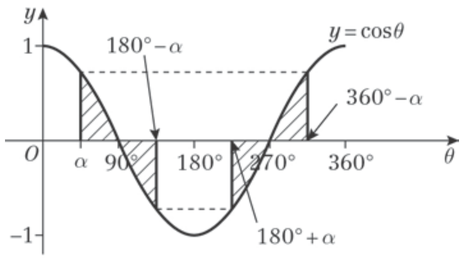
16 a

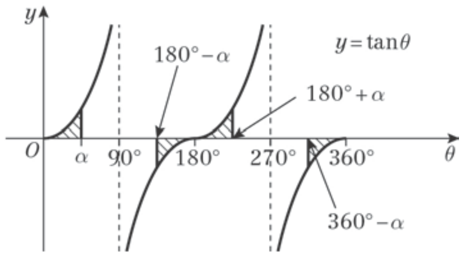


b The four shaded regions are congruent therefore the magnitude of the y value is the same for sin α.
Sin α° and sin $(108 – \alpha)°$ have the same y value (call it k).

16 b So $\sin \alpha° = \sin(180 – \alpha)°$, sin(180 \+ α)° and $\sin(360 – \alpha)°$ have the same y value, which will be $–k$.
So $\sin\alpha° = \sin(180 – \alpha)°$
$= –\sin(180 + \alpha)°$
$= –\sin(360 – \alpha)°$

17 a





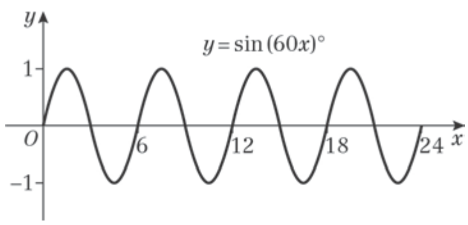
b i From the graph of $y = \cos \theta°$, which shows four congruent shaded regions, if the y value at $\alpha°$ is k, then y at $(180 – \alpha)°$ is $–k$, y at $(180 + \alpha)°$ is $–k$ and y at $(360 – \alpha)°$ it is $+k$.

So $\cos\alpha° = –\cos(180° – \alpha)$
$= –\cos(180° + \alpha)$
$= \cos(360° – \alpha)$

ii From the graph of $y = \tan \theta°$, if the y value at $\alpha°$ is k, then at $(180 – \alpha)°$ it is is $–k$, at $(180° + \alpha)$ it is $+k$ and at $(360° – \alpha)$ it is $–k$.

So $\tan \alpha° = –\tan(180° – \alpha)$
$= +\tan(180° + \alpha)$
$= −\tan(360° – \alpha)$

18 a



b There are 4 complete waves in the interval $0 \leq x \leq 24°$ so there are 4 sand dunes in this model.

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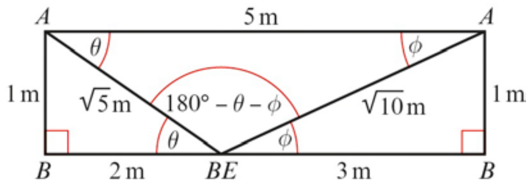
18 c The sand dunes may not all be the same height.

Challenge



$\angle ACB = \tan^{-1} 1 = 45°$

Show that $\theta + \phi = 45°$

$\sin\theta = \frac{1}{\sqrt{5}}$

Using the sine rule:

$$\frac{\sin(180° - \theta - \phi)}{5} = \frac{\sin\theta}{\sqrt{10}}$$

$$\sin(180° - \theta - \phi) = \frac{5\sin\theta}{\sqrt{10}}$$

Substituting $\sin\theta = \frac{1}{\sqrt{5}}$:

$$\sin(180° - \theta - \phi) = \frac{5\left(\frac{1}{\sqrt{5}}\right)}{\sqrt{10}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$, but angle $180° - \theta - \phi$ is obtuse.

So, $180° - \theta - \phi = 180° - 45° = 135°$

Therefore, $\theta + \phi = 45°$

So, $\angle AEB + \angle ADB = \angle ACB$

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