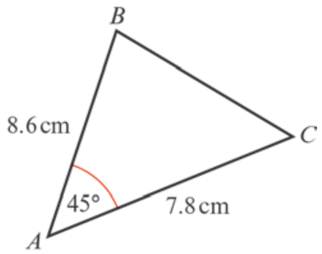
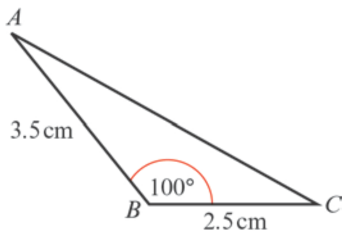
Trigonometric ratios 9D

1 a



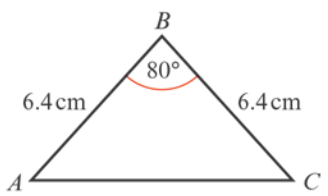
$$\text{Area} = \tfrac{1}{2} \times 7.8 \times 8.6 \times \sin 45°$$
$$= 23.71\ldots$$
$$= 23.7 \text{ cm}^2 \text{ (3 s.f.)}$$

b



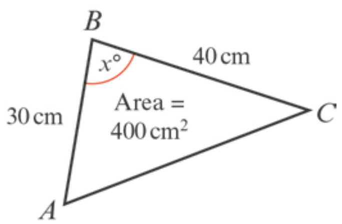
$$\text{Area} = \tfrac{1}{2} \times 2.5 \times 3.5 \times \sin 100°$$
$$= 4.308\ldots$$
$$= 4.31 \text{ cm}^2 \text{ (3 s.f.)}$$

c



$$\text{Area} = \tfrac{1}{2} \times 6.4 \times 6.4 \times \sin 80°$$
$$= 20.16\ldots$$
$$= 20.2 \text{ cm}^2 \text{ (3 s.f.)}$$

2 a

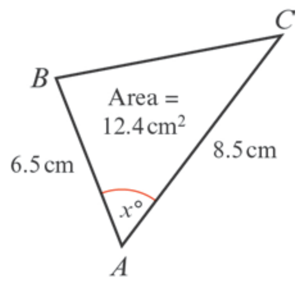


$$\text{Using area} = \tfrac{1}{2} ac \sin B$$
$$400 = \tfrac{1}{2} \times 40 \times 30 \times \sin x°$$

2 a So $\sin x° = \dfrac{400}{600} = \dfrac{2}{3}$

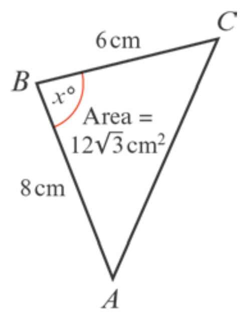
$$x° = \sin^{-1}\left(\tfrac{2}{3}\right) \text{ or } x° = 180° - \sin^{-1}\left(\tfrac{2}{3}\right)$$
$$x = 41.8 \text{ (3 s.f.) or } x = 138 \text{ (3 s.f.)}$$

b



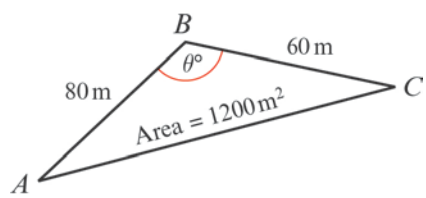
$$\text{Using area} = \tfrac{1}{2} bc \sin A$$
$$12.4 = \tfrac{1}{2} \times 8.5 \times 6.5 \times \sin x°$$
$$\text{So } \sin x° = \frac{12.4}{27.625} = 0.04488\ldots$$
$$x = 26.7 \text{ (3 s.f) or } x = 153 \text{ (3 s.f.)}$$

c



$$\text{Using area} = \tfrac{1}{2} ac \sin B$$
$$12\sqrt{3} = \tfrac{1}{2} \times 6 \times 8 \sin x°$$
$$\text{So } \sin x° = \frac{12\sqrt{3}}{24} = \frac{\sqrt{3}}{2}$$
$$x = 60 \text{ or } x = 120$$

3



$$\text{Using area} = \tfrac{1}{2} ac \sin B$$
$$1200 = \tfrac{1}{2} \times 60 \times 80 \times \sin \theta°$$

3 So $\sin\theta° = \frac{1200}{2400} = \frac{1}{2}$

$$\theta = 30 \text{ or } \theta = 150$$

But as AC is the largest side, θ must be the largest angle.
So $\theta = 150$
Using the cosine rule to find AC:

$b^2 = a^2 + c^2 - 2ac\cos B$

$AC^2 = 60^2 + 80^2 - 2\times 60\times 80\times \cos 150°$

$= 18\,313.84\ldots$

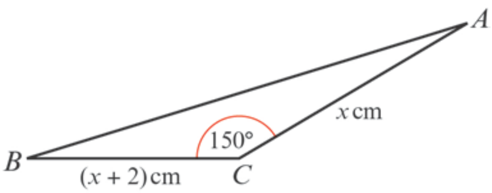
$AC = 135.3\ldots$

$AC = 135 \text{ m } (3 \text{ s.f.})$

So the perimeter $= 60 + 80 + 135$

$= 275\text{m } (3 \text{ s.f.})$

4



Area of $\Delta ABC = \frac{1}{2}x(x+2)\sin 150° \text{cm}^2$

So $5 = \frac{1}{2}x(x+2)\times\frac{1}{2}$

So $20 = x(x+2)$

$\Rightarrow x^2 + 2x - 20 = 0$

Using the quadratic formula:

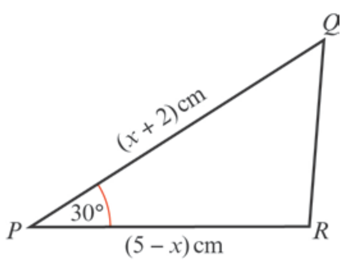
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2\pm\sqrt{84}}{2}$$

$x = 3.582\ldots$ or $x = -5.582\ldots$

As $x > 0$, $x = 3.58\ (3 \text{ s.f.})$

5



a Using area of $\Delta PQR = \frac{1}{2}qr\sin P$:

$A \text{ cm}^2 = \frac{1}{2}(5-x)(x+2)\sin 30° \text{ cm}^2$

5 a $\Rightarrow A = \frac{1}{2}(5x^2 - 2x + 10 - x^2)\times\frac{1}{2}$

$\Rightarrow A = \frac{1}{4}(10 + 3x - x^2)$

b Completing the square:

$10 + 3x - x^2 = -\left(\left(x-1\frac{1}{2}\right)^2 - 2\frac{1}{4} - 10\right)$

$= -\left(\left(x-1\frac{1}{2}\right)^2 - 12\frac{1}{4}\right)$

$= 12\frac{1}{4} - \left(x-1\frac{1}{2}\right)^2$

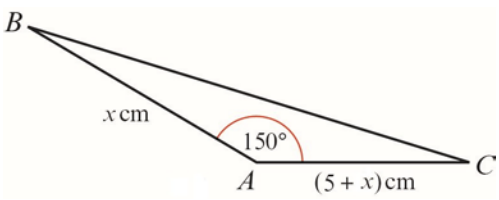
When $x = 1\frac{1}{2}$:

The maximum values of $10 + 3x - x^2 = 12\frac{1}{4}$ and the maximum value of A is

$\frac{1}{4}\left(12\frac{1}{4}\right) = 3\frac{1}{16}$

(You could use differentiation to find the maximum.)

6



a Using area of $\Delta BAC = \frac{1}{2}bc\sin A$

$3\frac{3}{4}\text{cm}^2 = \frac{1}{2}x(5+x)\sin 150° \text{ cm}^2$

$3\frac{3}{4} = \frac{1}{2}(5x + x^2)\times\frac{1}{2}$

$\Rightarrow 15 = 5x + x^2$

$\Rightarrow x^2 + 5x - 15 = 0$

b Using the quadratic equation formula:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-5\pm\sqrt{85}}{2}$$

$x = 2.109\ldots$ or $x = -7.109\ldots$

As $x > 0$, $x = 2.11\ (3 \text{ s.f.})$