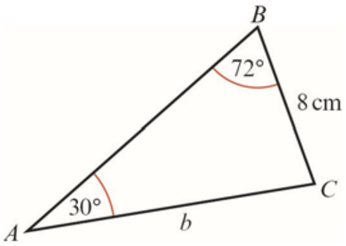
Trigonometric ratios 9B

1 a



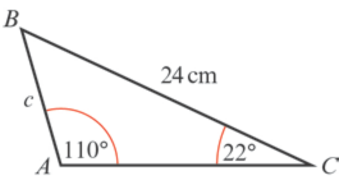
Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{b}{\sin 72°} = \frac{8}{\sin 30°}$$

$$\Rightarrow b = \frac{8 \sin 72°}{\sin 30°} = 15.2 \text{ cm } (3 \text{ s.f.})$$

$(\text{As } 72° > 30°,\ b > 8 \text{ cm})$

b



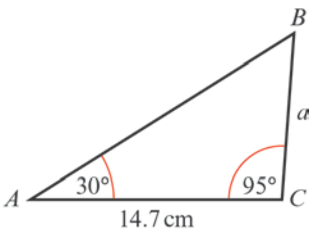
Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{c}{\sin 22°} = \frac{24}{\sin 110°}$$

$$\Rightarrow c = \frac{24 \sin 22°}{\sin 110°} = 9.57 \text{ cm } (3 \text{ s.f.})$$

$(\text{As } 110° > 22°,\ 24 \text{ cm} > c.)$

c



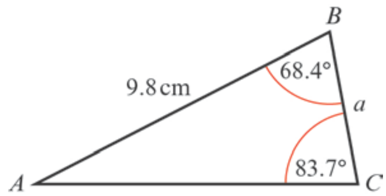
$$\angle ABC = 180° - (30 + 95)°$$
$$= 55°$$

Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{a}{\sin 30°} = \frac{14.7}{\sin 55°}$$

$$\Rightarrow a = \frac{14.7 \sin 30°}{\sin 55°} = 8.97 \text{ cm } (3 \text{ s.f.})$$

d



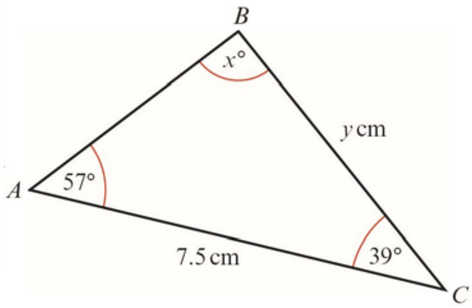
$$\angle ABC = 180° - (68.4 + 83.7)°$$
$$= 27.9°$$

Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{a}{\sin 27.9°} = \frac{9.8}{\sin 83.7°}$$

$$\Rightarrow a = \frac{9.8 \sin 27.9°}{\sin 83.7°} = 4.61 \text{ cm } (3 \text{ s.f.})$$

2 a



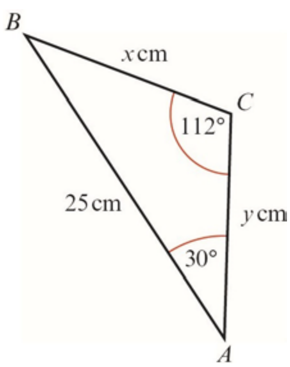
$$x = 180° - (57 + 39)°$$
$$= 84°$$

Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{y}{\sin 57°} = \frac{7.5}{\sin 84°}$$

$$\Rightarrow y = \frac{7.5 \sin 57°}{\sin 84°} = 6.32 \text{ cm } (3 \text{ s.f.})$$

b



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

2 b $\frac{x}{\sin 30°} = \frac{25}{\sin 112°}$

$\Rightarrow x = \frac{25\sin 30°}{\sin 112°} = 13.5 \text{ cm } (3 \text{ s.f.})$

$\angle B = 180° - (112 + 30)°$

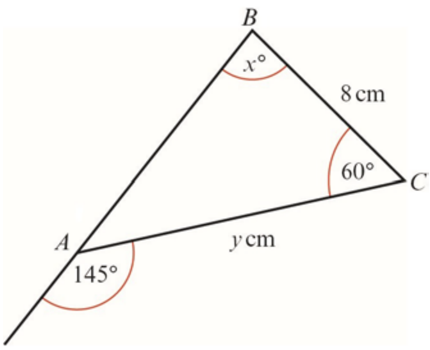
$= 38°$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{y}{\sin 38°} = \frac{25}{\sin 112°}$

$\Rightarrow y = \frac{25\sin 38°}{\sin 112°} = 16.6 \text{ cm } (3 \text{ s.f.})$

c



$x = 180° - (60 + 35)°$

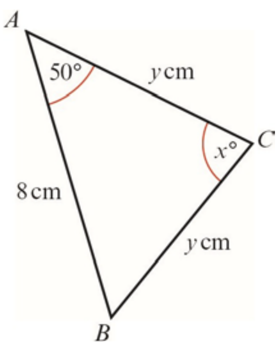
$= 85°$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{y}{\sin 85°} = \frac{8}{\sin 35°}$

$\Rightarrow y = \frac{8\sin 85°}{\sin 35°} = 13.9 \text{ cm } (3 \text{ s.f.})$

d



$x = 180° - (50 + 50)°$

$= 80°$

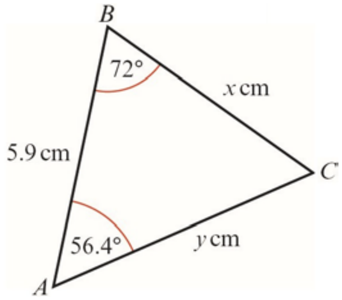
Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

d $\frac{y}{\sin 50°} = \frac{8}{\sin 80°}$

$\Rightarrow y = \frac{8\sin 50°}{\sin 80°} = 6.22 \text{ cm } (3 \text{ s.f.})$

(Note: You could use the line of symmetry to split the triangle into two right-angled triangles and use $\cos 50° = \frac{4}{y}$.)

e



$\angle C = 180° - (56.4 + 72)°$

$= 51.6°$

Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{x}{\sin 56.4°} = \frac{5.9}{\sin 51.6°}$

$\Rightarrow x = \frac{5.9\sin 56.4°}{\sin 51.6°}$

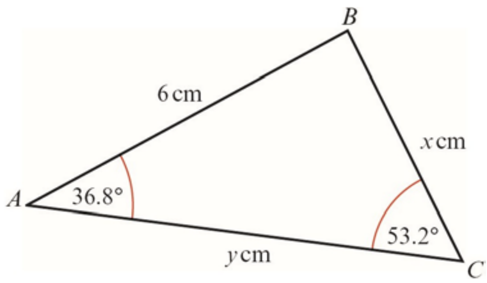
$= 6.27 \text{ cm } (3 \text{ s.f.})$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{y}{\sin 72°} = \frac{5.9}{\sin 51.6°}$

$\Rightarrow y = \frac{5.9\sin 72°}{\sin 51.6°} = 7.16 \text{ cm } (3 \text{ s.f.})$

f



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{x}{\sin 36.8°} = \frac{6}{\sin 53.2°}$

2 f $\Rightarrow x = \dfrac{6\sin 36.8°}{\sin 53.2°} = 4.49 \text{ cm } (3 \text{ s.f.})$

$\angle B = 180° - (36.8 + 53.2)°$

$= 90°$

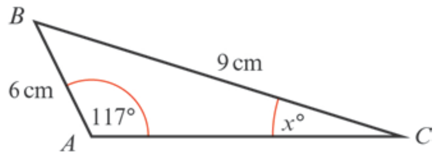
Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{6}{\sin 53.2°} = \dfrac{y}{\sin 90°}$

$\Rightarrow y = \dfrac{6\sin 90°}{\sin 53.2°} = 7.49 \text{ cm } (3 \text{ s.f.})$

(Note: The third angle is 90° so you could solve the problem using sine or cosine; the sine rule is not necessary.)

3 a



Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

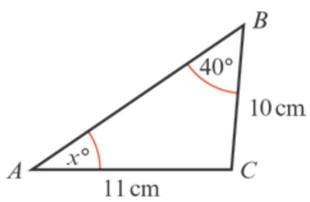
$\dfrac{\sin x°}{6} = \dfrac{\sin 117°}{9}$

$\Rightarrow \sin x° = \dfrac{6\sin 117°}{9} \; (= 0.5940\ldots)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{6\sin 117°}{9}\right) = 36.4° \; (3 \text{ s.f.})$

$\Rightarrow x = 36.4$

b



Using $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

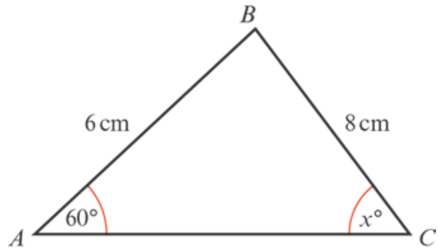
$\dfrac{\sin x°}{10} = \dfrac{\sin 40°}{11}$

$\Rightarrow \sin x° = \dfrac{10\sin 40°}{11} \; (= 0.5843\ldots)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{10\sin 40°}{11}\right) = 35.8° \; (3 \text{ s.f.})$

$\Rightarrow x = 35.8$

3 c



Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

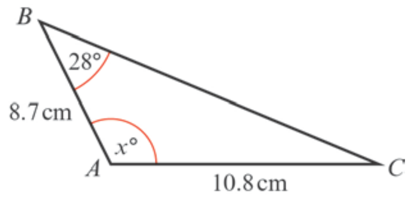
$\dfrac{\sin x°}{6} = \dfrac{\sin 60°}{8}$

$\Rightarrow \sin x° = \dfrac{6\sin 60°}{8} \; (= 0.6495\ldots)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{6\sin 60°}{8}\right) = 40.5° \; (3 \text{ s.f.})$

$\Rightarrow x = 40.5$

d



Using $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{8.7} = \dfrac{\sin 28°}{10.8}$

$\Rightarrow \sin C = \dfrac{8.7\sin 28°}{10.8} \; (= 0.3781\ldots)$

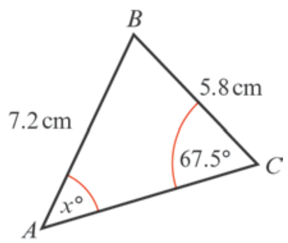
$\Rightarrow C = \sin^{-1}\left(\dfrac{8.7\sin 28°}{10.8}\right)$

$\Rightarrow C = 22.2° \; (3 \text{ s.f.})$

$\Rightarrow x° = 180° - (28 + 22.2)° = 130° \; (3 \text{ s.f.})$

$\Rightarrow x = 130$

4 a



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

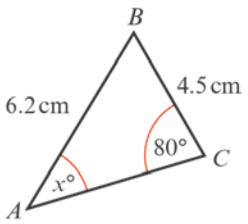
$\dfrac{\sin x°}{5.8} = \dfrac{\sin 67.5°}{7.2}$

4 a $\Rightarrow \sin x° = \dfrac{5.8\sin 67.5°}{7.2} (= 0.7442...)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{5.8\sin 67.5°}{7.2}\right) = 48.09°$

$\Rightarrow x = 48.1$ (3 s.f.)

b



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

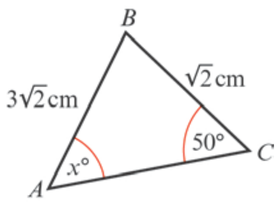
$\dfrac{\sin x°}{4.5} = \dfrac{\sin 80°}{6.2}$

$\Rightarrow \sin x° = \dfrac{4.5\sin 80°}{6.2} (= 0.7147...)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{4.5\sin 80°}{6.2}\right) = 45.63°$

$\Rightarrow x = 45.6$ (3 s.f.)

c



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

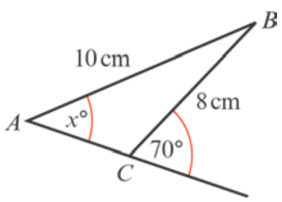
$\dfrac{\sin x°}{\sqrt{2}} = \dfrac{\sin 50°}{3\sqrt{2}}$

$\Rightarrow \sin x° = \dfrac{\sqrt{2}\sin 50°}{3\sqrt{2}} (= 0.2553...)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{\sin 50°}{3}\right) = 14.79°$

$\Rightarrow x = 14.8$ (3 s.f.)

d



Angle $ACB = 180° - 70° = 110°$

Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x°}{8} = \dfrac{\sin 110°}{10}$

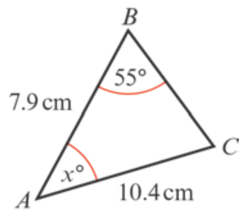
$\Rightarrow \sin x° = \dfrac{8\sin 110°}{10} (= 0.7517...)$

$\Rightarrow x° = \sin^{-1}\left(\dfrac{8\sin 110°}{10}\right)$

$= 48.74°$

$\Rightarrow x = 48.7$ (3 s.f.)

e



Using $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{7.9} = \dfrac{\sin 55°}{10.4}$

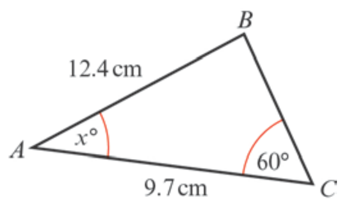
$\Rightarrow \sin C = \dfrac{7.9\sin 55°}{10.4} (= 0.6222...)$

$\Rightarrow C = \sin^{-1}\left(\dfrac{7.9\sin 55°}{10.4}\right) = 38.48°$

$x° = 180° - (55° + C)$

$\Rightarrow x = 86.52 = 86.5$ (3 s.f.)

f



Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin B}{9.7} = \dfrac{\sin 60°}{12.4}$

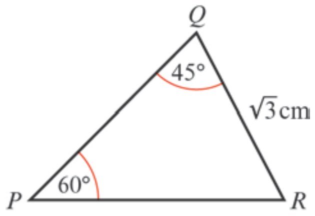
$\Rightarrow \sin B = \dfrac{9.7\sin 60°}{12.4} (= 0.6774...)$

$\Rightarrow B = 42.65°$

$x° = 180° - (60 + B)° = 77.35°$

$\Rightarrow x = 77.4$ (3 s.f.)

5



a Using $\dfrac{q}{\sin Q} = \dfrac{p}{\sin P}$

$$\frac{PR}{\sin 45^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow PR = \frac{\sqrt{3}\sin 45^\circ}{\sin 60^\circ} = 1.41 \text{ cm } (3 \text{ s.f.})$$

(The exact answer is $\sqrt{2}$ cm.)

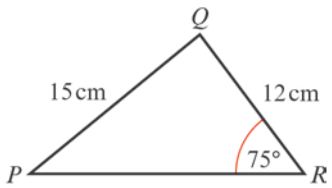
b Using $\dfrac{r}{\sin R} = \dfrac{p}{\sin P}$

$$(R = 180^\circ - (60 + 45)^\circ = 75^\circ)$$

$$\frac{PQ}{\sin 75^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow PQ = \frac{\sqrt{3}\sin 75^\circ}{\sin 60^\circ} = 1.93 \text{ cm } (3 \text{ s.f.})$$

6



Using $\dfrac{\sin P}{p} = \dfrac{\sin R}{r}$

$$\frac{\sin P}{12} = \frac{\sin 75^\circ}{15}$$

$$\Rightarrow \sin P = \frac{12\sin 75^\circ}{15} (= 0.7727\ldots)$$

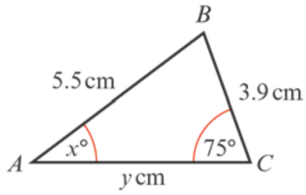
$$\Rightarrow P = \sin^{-1}\left(\frac{12\sin 75^\circ}{15}\right) = 50.60^\circ$$

Angle $QPR = 50.6^\circ$ (3 s.f.)

Angle $PQR = 180^\circ - (75 + 50.6)^\circ$

$= 54.4^\circ$ (3 s.f.)

7 a



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin x^\circ}{3.9} = \frac{\sin 75^\circ}{5.5}$$

$$\Rightarrow \sin x^\circ = \frac{3.9\sin 75^\circ}{5.5}$$

$$\Rightarrow x^\circ = \sin^{-1}\left(\frac{3.9\sin 75^\circ}{5.5}\right)$$

$$= 43.23^\circ$$

$$\Rightarrow x = 43.2 \text{ (3 s.f.)}$$

So $\angle ABC = 180^\circ - (75 + 43.2)^\circ = 61.8^\circ$

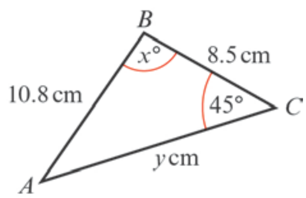
Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\frac{y}{\sin 61.8^\circ} = \frac{5.5}{\sin 75^\circ}$$

$$\Rightarrow y = \frac{5.5\sin 61.8^\circ}{\sin 75^\circ} = 5.018$$

$$\Rightarrow y = 5.02 \text{ (3 s.f.)}$$

b



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin A}{8.5} = \frac{\sin 45^\circ}{10.8}$$

$$\Rightarrow \sin A = \frac{8.5\sin 45^\circ}{10.8}$$

$$\Rightarrow A = \sin^{-1}\left(\frac{8.5\sin 45^\circ}{10.8}\right) = 33.815^\circ$$

$$x^\circ = 180^\circ - (45^\circ + A) = 101.2^\circ$$

$$\Rightarrow x = 101 \text{ (3 s.f.)}$$

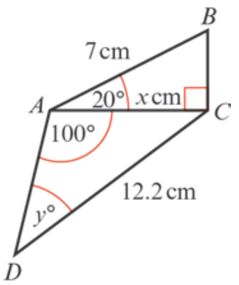
7 b Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{y}{\sin x°} = \frac{10.8}{\sin 45°}$

$\Rightarrow y = \frac{10.8 \sin x°}{\sin 45°} = 14.98$

$\Rightarrow y = 15.0\ (3\text{ s.f.})$

c



In ΔABC, $\frac{x}{7} = \cos 20°$

$\Rightarrow x = 7\cos 20°$

$= 6.58\ (3\text{ s.f.})$

Using $\frac{\sin D}{d} = \frac{\sin A}{a}$ in ΔADC

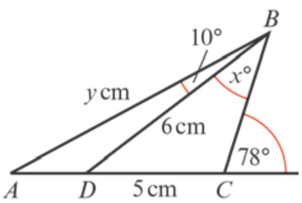
$\frac{\sin y°}{x} = \frac{\sin 100°}{12.2}$

$\Rightarrow \sin y° = \frac{x \sin 100°}{12.2}$

$\Rightarrow y° = \sin^{-1}\left(\frac{x\sin 100°}{12.2}\right) = 32.07°$

$\Rightarrow y = 32.1\ (3\text{ s.f.})$

d



In triangle *BDC*:

$\angle C = 180° - 78° = 102°$

Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin x°}{5} = \frac{\sin 102°}{6}$

$\Rightarrow \sin x° = \frac{5\sin 102°}{6}$

d $\Rightarrow x° = \sin^{-1}\left(\frac{5\sin 102°}{6}\right) = 54.599°$

$\Rightarrow x = 54.6\ (3\text{ s.f.})$

In triangle *ABC*:

$\angle BAC = 180° - 102° - (10 + x)° = 13.4°$

So $\angle ADB = 180° - 10° - 13.4° = 156.6°$

Using $\frac{d}{\sin D} = \frac{a}{\sin A}$ in ΔABD

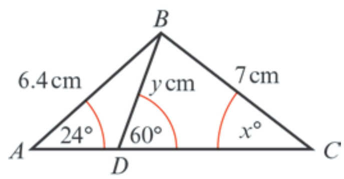
$\frac{y}{\sin 156.6°} = \frac{6}{\sin 13.4°}$

$\Rightarrow y = \frac{6\sin 156.6°}{\sin 13.4°}$

$= 10.28$

$= 10.3\ (3\text{ s.f.})$

e



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$ in ΔABC

$\frac{\sin x°}{6.4} = \frac{\sin 24°}{7}$

$\Rightarrow x = 21.8\ (3\text{ s.f.})$

Using $\frac{a}{\sin A} = \frac{d}{\sin D}$ in ΔABD

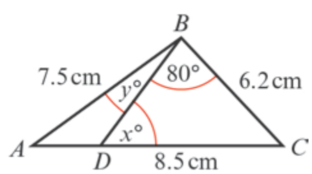
$\frac{y}{\sin 24°} = \frac{6.4}{\sin 120°}$

$\Rightarrow y = \frac{6.4\sin 24°}{\sin 120°} = 3.0058$

$\Rightarrow y = 3.01\ (3\text{ s.f.})$

(The above approach finds the two values independently. You could find *y* first and then use it to find *x*, but if your answer for *y* is wrong then *x* will be wrong as well.)

f



7 f Using $\frac{\sin D}{d}=\frac{\sin B}{b}$ in ΔBDC

$$\frac{\sin x°}{6.2}=\frac{\sin 80°}{8.5}$$

$$\Rightarrow \sin x°=\frac{6.2\sin 80°}{8.5}$$

$$\Rightarrow x°=\sin^{-1}\left(\frac{6.2\sin 80°}{8.5}\right)=45.92°$$

$$\Rightarrow x=45.9\ (3\text{ s.f.})$$

In triangle ABC:

$$\angle ACB=180°-(80+x)°$$
$$=54.08°$$

Using $\frac{\sin A}{a}=\frac{\sin C}{c}$

$$\frac{\sin A}{6.2}=\frac{\sin 54.08°}{7.5}$$

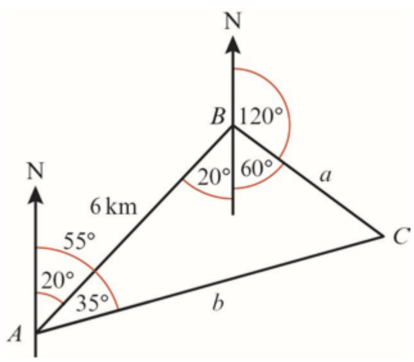
$$\Rightarrow \sin A=\frac{6.2\sin 54.08°}{7.5}$$

$$\Rightarrow A=\sin^{-1}\left(\frac{6.2\sin 54.08°}{7.5}\right)$$
$$=42.03°$$

So $y°=180°-(42.03+134.1)°$

$y=3.87\ (3\text{ s.f.})$

8



$\angle BAC=55°-20°=35°$

$\angle ABC=20°+60°=80°$

(Alternate angles and angles on a straight line.)

$\angle ACB=180°-(80+35)°=65°$

a Using $\frac{b}{\sin B}=\frac{c}{\sin C}$

$$\Rightarrow \frac{AC}{\sin 80°}=\frac{6}{\sin 65°}$$

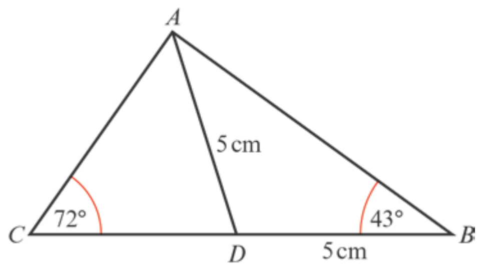
$$\Rightarrow AC=\frac{6\sin 80°}{\sin 65°}=6.52\text{ km }(3\text{ s.f.})$$

8 b Using $\frac{a}{\sin A}=\frac{c}{\sin C}$

$$\Rightarrow \frac{BC}{\sin 35°}=\frac{6}{\sin 65°}$$

$$\Rightarrow BC=\frac{6\sin 35°}{\sin 65°}=3.80\text{ km }(3\text{ s.f.})$$

9



a In triangle ABD:

$\angle DAB=43°$ (isosceles Δ)

So $\angle ADB=180°-(2\times 43°)=94°$

As the triangle is isosceles you could work with right-angled triangles, but using the sine rule

$$\frac{d}{\sin D}=\frac{a}{\sin A}$$

$$\Rightarrow \frac{AB}{\sin 94°}=\frac{5}{\sin 43°}$$

$$\Rightarrow AB=\frac{5\sin 94°}{\sin 43°}=7.31\text{cm }(3\text{ s.f.})$$

b In triangle ADC:

$\angle ADC=180°-94°=86°$

So $\angle CAD=180°-(72+86)°=22°$

Using $\frac{a}{\sin A}=\frac{c}{\sin C}$

$$\Rightarrow \frac{CD}{\sin 22°}=\frac{5}{\sin 72°}$$

$$\Rightarrow CD=\frac{5\sin 22°}{\sin 72°}=1.97\text{ cm }(3\text{ s.f.})$$

10 a In triangle ABD:

$$\frac{\sin B}{b}=\frac{\sin A}{a}$$

$$\frac{\sin B}{76}=\frac{\sin 66°}{136}$$

So $\sin B=\frac{76\sin 66°}{136}$

$B=30.6978...°$

10 a So the angle between *AB* and *BD* is 30.7°.
Using triangle BCD:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{80} = \frac{\sin 98°}{136}$$

So $\sin B = \frac{80 \sin 98°}{136}$

$B = 35.6273...°$

So the angle between *BC* and *BD* is 35.6°.
The angle between the fences *AB* and *BC* is 30.7° + 35.6° = 66.3°.

b In triangle *ABD*:
Angle *ADB* = 180° − 66° − 30.7° = 83.3°
Using the cosine rule:

$$d^2 = a^2 + b^2 - 2ab\cos D$$

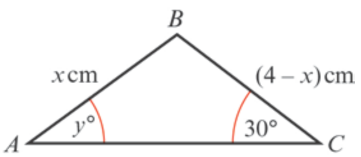
$$d^2 = 136^2 + 76^2 - 2 \times 136 \times 76 \cos 83.3°$$

$$d^2 = 18\,496 + 5776 - 2411.817\,477$$

$$d^2 = 21\,860.182\,52$$

So $d = 147.851...$
So the length of the fence *AB* is 148 m (3 s.f.).

11



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{4-x}{\sin y°} = \frac{x}{\sin 30°}$$

$$\Rightarrow (4-x)\sin 30° = x \sin y°$$

$$\Rightarrow (4-x) \times \frac{1}{2} = x \times \frac{1}{\sqrt{2}}$$

Multiply throughout by 2:

$$4 - x = x\sqrt{2}$$

$$x + \sqrt{2}x = 4$$

$$x(1+\sqrt{2}) = 4$$

$$x = \frac{4}{1+\sqrt{2}}$$

11 Multiply 'top and bottom' by $\sqrt{2}-1$:

$$x = \frac{4(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{4(\sqrt{2}-1)}{2-1}$$

$$= 4(\sqrt{2}-1)$$

12 a Using the left-hand triangle, the angles are 40°, 128° and 12°. (128° = 180° − 52° and 180° − (128° + 40°) = 12°)

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 128°} = \frac{15}{\sin 12°}$$

$$a = \frac{15 \sin 128°}{\sin 12°}$$

$a = 56.8518...$
Using the larger right-angled triangle:

$$\sin 40° = \frac{\text{height}}{56.8518}$$

Height = 56.8518 sin 40° = 36.54...
The height of the building is 36.5 m (3 s.f.).

b Assume that the angles of elevation have been measured from ground level.

