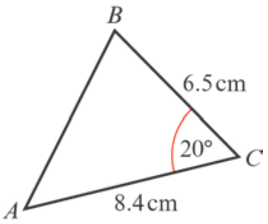
Trigonometric ratios 9A

1 a



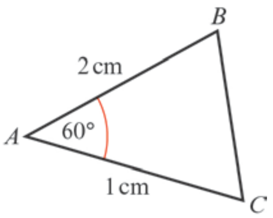
Using $c^2 = a^2 + b^2 - 2ab\cos C$

$AB^2 = 6.5^2 + 8.4^2 - 2\times 6.5\times 8.4\times \cos 20°$

$AB^2 = 10.1955\ldots$

$AB = \sqrt{10.1955\ldots} = 3.19\text{ cm } (3\text{ s.f.})$

b



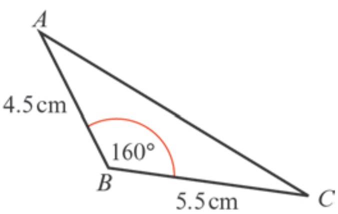
Using $a^2 = b^2 + c^2 - 2bc\cos A$

$BC^2 = 1^2 + 2^2 - 2\times 1\times 2\times \cos 60°$

$BC^2 = 3$

$BC = \sqrt{3} = 1.73\text{ cm } (3\text{ s.f.})$

c



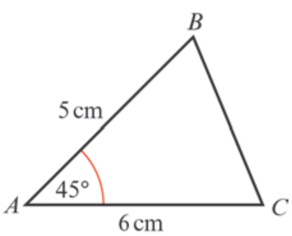
Using $b^2 = a^2 + c^2 - 2ac\cos B$

$AC^2 = 5.5^2 + 4.5^2 - 2\times 5.5\times 4.5\times \cos 160°$

$AC^2 = 97.014\ldots$

$AC = \sqrt{97.014\ldots3} = 9.85\text{ cm } (3\text{ s.f.})$

d



Using $a^2 = b^2 + c^2 - 2bc\cos A$

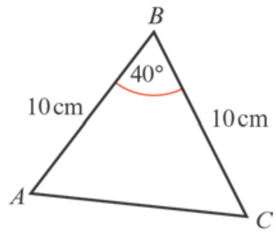
d $BC^2 = 6^2 + 5^2 - 2\times 6\times 5\times \cos 45°$

$= 18.573\ldots$

$BC = \sqrt{18.573\ldots}$

$= 4.31\text{ cm } (3\text{ s.f.})$

e



(This is an isosceles triangle and so you could use right-angled triangle work.)

Using $b^2 = a^2 + c^2 - 2ac\cos B$

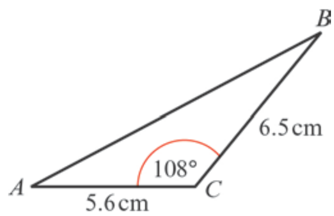
$AC^2 = 10^2 + 10^2 - 2\times 10\times 10\times \cos 40°$

$= 46.791\ldots$

$AC = \sqrt{46.791\ldots}$

$= 6.84\text{cm } (3\text{ s.f.})$

f



Using $c^2 = a^2 + b^2 - 2ab\cos C$

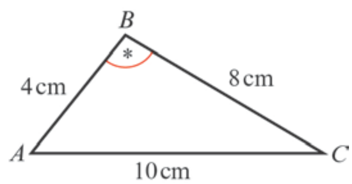
$AB^2 = 6.5^2 + 5.6^2 - 2\times 6.5\times 5.6\times \cos 108°$

$= 96.106\ldots$

$AB = \sqrt{96.106\ldots}$

$= 9.80\text{ cm } (3\text{ s.f.})$

2 a



Using $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{8^2 + 4^2 - 10^2}{2\times 8\times 4}$

2 **a** $\cos B = -\frac{20}{64}$

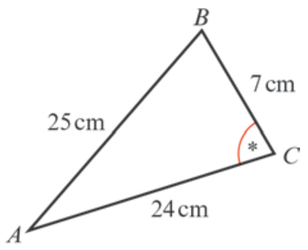
$= -\frac{5}{16}$

$B = \cos^{-1}\left(-\frac{5}{16}\right) = 108.2\ldots°$

$= 108°\ (3\text{ s.f.})$

We can use a calculator to find directly an obtuse angle with a negative cosine value.

b



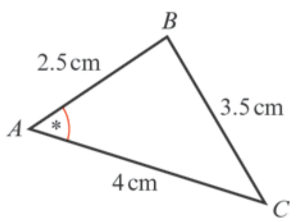
Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{7^2 + 24^2 - 25^2}{2 \times 7 \times 24}$

$= 0$

$\Rightarrow C = 90°$

c



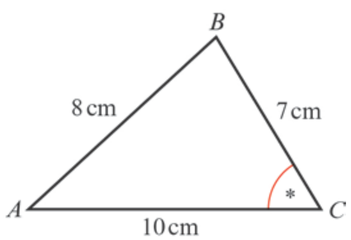
Using $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{4^2 + 2.5^2 - 3.5^2}{2 \times 4 \times 2.5}$

$= \frac{1}{2}$

$A = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

d



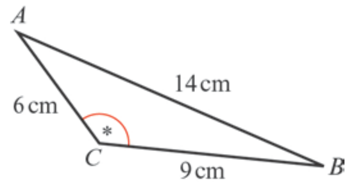
Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

d $\cos C = \dfrac{7^2 + 10^2 - 8^2}{2 \times 7 \times 10}$

$= 0.6071$

$\Rightarrow C = 52.6°\ (3\text{ s.f.})$

e



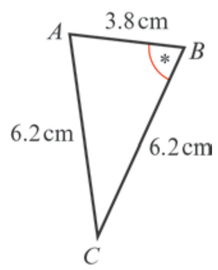
Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{9^2 + 6^2 - 14^2}{2 \times 9 \times 6}$

$= -0.7314\ldots$

$\Rightarrow C = 137°\ (3\text{ s.f.})$

f



(This is an isosceles triangle so you could use right-angled triangle trigonometry.)

Using $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{6.2^2 + 3.8^2 - 6.2^2}{2 \times 6.2 \times 3.8}$

$= 0.3064\ldots$

$\Rightarrow B = 72.2°\ (3\text{ s.f.})$

3 Use alternate angles to find angle of 40° and 180°−130° = 50°. Adding, this gives 90°. At this point, you can use Pythagoras' theorem or the cosine rule.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 120^2 + 150^2 - 2 \times 120 \times 150 \cos 90°$

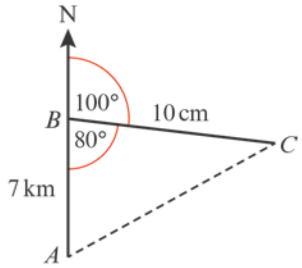
$= 14\,400 + 22\,500 - 0$

$= 36\,900$

So c = 192.0937...

So the distance of the plane from the airport is 192 km (3 s.f.).

4



Using the cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos B$

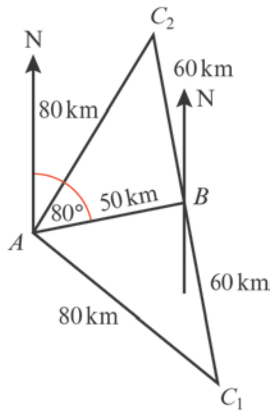
$AC^2 = 10^2 + 7^2 - 2\times 10\times 7\times \cos 80°$

$= 124.689$

$AC = \sqrt{124.689\ldots}$

$= 11.2\ \text{km}\ (3\text{ s.f.})$

5



The bearing of C from B is not given so there are two possibilities for C, using the data.
The angle A will be the same in each ΔABC.

Using $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

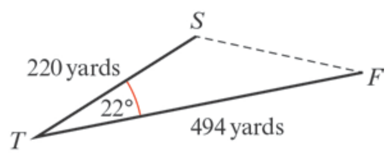
$\cos A = \frac{80^2 + 50^2 - 60^2}{2\times 80\times 50} = 0.6625$

$\Rightarrow A = 48.5°$

The bearing of C from A is

$80° \pm 48.5° = 128.5°$ or $031.5°$

6



Using the cosine rule:

$t^2 = f^2 + s^2 - 2fs\cos T$

6 $SF^2 = 220^2 + 494^2 - 2\times 220\times 494\cos 22°$

$= 90\,903.317$

$SF = \sqrt{90\,903.317\ldots} = 301.5\ldots$ yards

$= 302$ yards (3 s.f.)

7 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{5^2 + 4^2 - 6^2}{2(5)(4)}$

$\cos A = \frac{25 + 16 - 36}{40}$

$\cos A = \frac{5}{40}$

$\cos A = \frac{1}{8}$

8 $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$

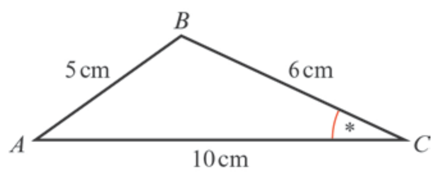
$\cos P = \frac{3^2 + 2^2 - 4^2}{2(3)(2)}$

$\cos P = \frac{9 + 4 - 16}{12}$

$\cos P = -\frac{3}{12}$

$\cos P = -\frac{1}{4}$

9



The smallest angle is C as this is opposite AB, the shortest side.

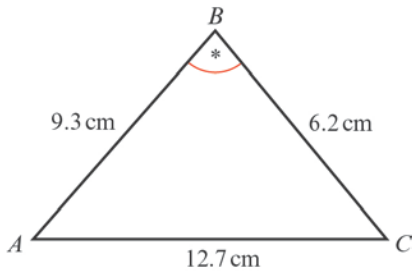
Using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{6^2 + 10^2 - 5^2}{2\times 6\times 10}$

$= 0.925$

$C = 22.3°\ (3\text{ s.f.})$

10



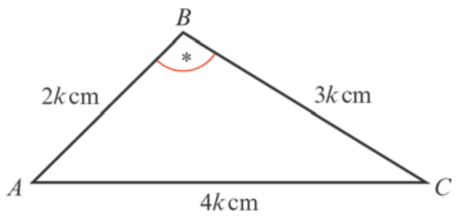
The largest angle is B as it is opposite AC.

$$\text{Using } \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{6.2^2 + 9.3^2 - 12.7^2}{2 \times 6.2 \times 9.3} = -0.3152\ldots$$

$$B = 108.37\ldots = 108° \text{ (3 s.f.)}$$

11



The largest angle will be opposite the side of length $4k$ cm, the longest side.

$$\text{Using } \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

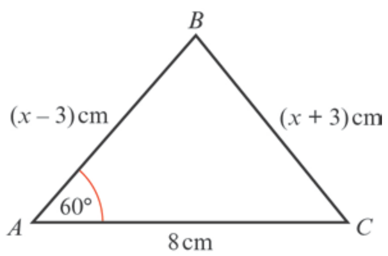
$$\cos B = \frac{9k^2 + 4k^2 - 16k^2}{2 \times 3k \times 2k}$$

$$= -0.25$$

$$B = 104.477\ldots$$

$$= 104° \text{ (3 s.f.)}$$

12



$$\text{Using } a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x+3)^2 = (x-3)^2 + 8^2 - 2 \times 8 \times (x-3)\cos 60°$$

$$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8(x-3)$$

$$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8x + 24$$

12

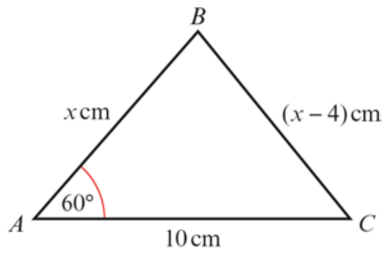
$$6x + 6x + 8x = 64 + 24$$

$$20x = 88$$

$$x = \frac{88}{20}$$

$$= 4.4 \text{ cm}$$

13



$$\text{Using } a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x-4)^2 = 10^2 + x^2 - 2 \times 10 \times x\cos 60°$$

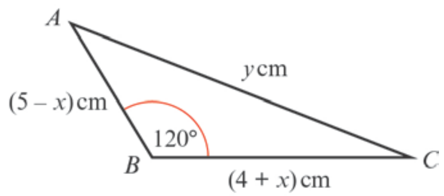
$$x^2 - 8x + 16 = 100 + x^2 - 10x$$

$$10x - 8x = 100 - 16$$

$$2x = 84$$

$$x = 42 \text{ cm}$$

14 a



Using $b^2 = a^2 + c^2 - 2ac\cos B$

$y^2 = (4 + x)^2 + (5 - x)^2 - 2(4 + x)(5 - x)$ $\cos 120°$

$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + (4 + x)$ $(5 - x)$

(Note : $2\cos 120° = -1$)

$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + 20 + x$ $- x^2$

$= x^2 - x + 61$

b Completing the square:

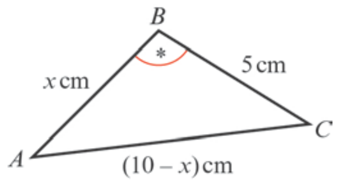
$$y^2 = \left(x - \tfrac{1}{2}\right)^2 + 61 - \tfrac{1}{4}$$

$$\Rightarrow y^2 = \left(x - \tfrac{1}{2}\right)^2 + 60\tfrac{3}{4}$$

The minimum value of y^2 occurs when $\left(x - \tfrac{1}{2}\right)^2 = 0$ i.e. when $x = \tfrac{1}{2}$.

So the minimum value of y^2 is 60.75.

15 a



$$\cos B = \frac{5^2 + x^2 - (10-x)^2}{2\times 5\times x}$$
$$= \frac{25 + x^2 - (100 - 20x + x^2)}{10x}$$
$$= \frac{25 + x^2 - 100 + 20x - x^2}{10x}$$
$$= \frac{20x - 75}{10x}$$
$$= \frac{5(4x-15)}{10x}$$
$$= \frac{4x-15}{2x}$$

b As $\cos B = -\frac{1}{7}$

$$\frac{4x-15}{2x} = -\frac{1}{7}$$
$$7(4x-15) = -2x$$
$$28x - 105 = -2x$$
$$30x = 105$$
$$x = \frac{105}{30}$$
$$= 3\tfrac{1}{2}$$

16 First find the length of the diagonal *BD*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 120^2 + 75^2 - 2\times 120\times 75\cos 74°$$
$$a^2 = 14\,400 + 5625 - 4961.4724$$
$$a^2 = 15\,063.5276$$

So a = 122.733 56...

So the length of the diagonal *BD* is 122.733 56... m.

Note that in this question you do not have to find the value of a since you only need a^2 in the next part of the calculation.

16 To find the angle between fences *BC* and *CD*:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{135^2 + 60^2 - 122.733\,56^2}{2(135)(60)}$$
$$\cos C = \frac{18\,225 + 3600 - 15\,063.5276}{16\,200}$$
$$\cos C = 0.417\,37...$$
$$C = \cos^{-1} 0.417\,37...$$
$$= 65.33...°$$

So the angle between fences *BC* and *CD* is 65.3° (3 s.f.).

17 a

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 70^2 + 50^2 - 2\times 70\times 50\cos 20°$$
$$a^2 = 4900 + 2500 - 6577.848...$$
$$a^2 = 822.151\,65...$$

So a = 28.673...

So the distance between ships *B* and *C* is 28.7 km (3 s.f.).

b

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{28.673^2 + 50^2 - 70^2}{2(28.673)(50)}$$
$$\cos B = \frac{822.151\,65 + 2500 - 4900}{2867.3187}$$
$$\cos B = -0.550\,28...$$
$$B = \cos^{-1} 0.550\,28...$$
$$= 123.3867...°$$

The bearing is 180° − 123.3867° = 56.6°.

So the bearing of ship *C* from ship *B* is 056.6°.

