The binomial expansion 8E

1 a $\left(1-\frac{x}{10}\right)^6 = 1^6 + \binom{6}{1}1^5\left(-\frac{x}{10}\right) + \binom{6}{2}1^4\left(-\frac{x}{10}\right)^2 + \binom{6}{3}1^3\left(-\frac{x}{10}\right)^3 + \ldots$

$= 1 - 0.6x + 0.15x^2 - 0.02x^3 + \ldots$

b We want $(1 - \frac{x}{10}) = 0.99$

$\frac{x}{10} = 0.01$

$x = 0.1$

Substituting $x = 0.1$ into the expansion for $\left(1-\frac{x}{10}\right)^6$:

$0.99^6 \approx 1 - 0.6(0.1) + 0.15(0.1)^2 - 0.02(0.1)^3$

$\approx 0.941\,48$

2 a $\left(2+\frac{x}{5}\right)^{10} = 2^{10} + \binom{10}{1}2^9\left(\frac{x}{5}\right) + \binom{10}{2}2^8\left(\frac{x}{5}\right)^2 + \binom{10}{3}2^7\left(\frac{x}{5}\right)^3 + \ldots$

$= 1024 + 1024x + 460.8x^2 + 122.88x^3 + \ldots$

b We want $\left(2+\frac{x}{5}\right)^{10} = 2.1$

$\frac{x}{5} = 0.1$

$x = 0.5$

Substituting $x = 0.5$ into the expansion for $\left(2+\frac{x}{5}\right)^{10}$:

$2.1^{10} \approx 1024 + 1024(0.5) + 460.8(0.5)^2 + 122.88(0.5)^3$

≈ 1666.56

3 $(1 - 3x)^5 = 1^5 + \binom{5}{1}1^4(-3x) + \binom{5}{2}1^3(-3x)^2 + \ldots$

$= 1 - 15x + 90x^2 + \ldots$

$(2 + x)(1 - 3x)^5 = (2 + x)(1 - 15x + 90x^2 + \ldots)$

$= 2 - 30x + 180x^2 + \ldots + x - 15x^2 + \ldots$

$\approx 2 - 29x + 165x^2$

4 $(3 + x)^4 = 3^4 + \binom{4}{1}3^3x + \binom{4}{2}3^2x^2 + \ldots$

$= 81 + 108x + 54x^2 + \ldots$

$(2 - x)(3 + x)^4 = (2 - x)(81 + 108x + 54x^2 + \ldots)$

$= 162 + 216x + 108x^2 + \ldots - 81x - 108x^2 + \ldots$

$\approx 162 + 135x + 0x^2 + \ldots$

$a = 162$, $b = 135$, $c = 0$

5 a $(1+2x)^8 = 1^8 + \binom{8}{1}1^7(2x) + \binom{8}{2}1^6(2x)^2 + \binom{8}{3}1^5(2x)^3 + \ldots$
$= 1 + 16x + 112x^2 + 448x^3 + \ldots$

b We want $(1 + 2x) = 1.02$
$2x = 0.02$
$x = 0.01$

Substituting $x = 0.01$ into the expansion for $(1 + 2x)^8$:
$1.02^8 \approx 1 + 16(0.01) + 112(0.01)^2 + 448(0.01)^3$
$\approx 1.171\,648$

6 a $(1-5x)^{30} = 1^{30} + \binom{30}{1}1^{29}(-5x) + \binom{30}{2}1^{28}(-5x)^2 + \binom{30}{3}1^{27}(-5x)^3 + \ldots$
$= 1 - 150x + 10\,875x^2 - 507\,500x^3 + \ldots$

b We want $(1 - 5x) = 0.995$
$5x = 0.005$
$x = 0.001$

Substituting $x = 0.001$ into the expansion for $(1 - 5x)^{30}$
$0.995^{30} \approx 1 - 150(0.001) + 10\,875(0.001)^2 - 507\,500(0.001)^3$
$\approx 0.860\,368$

c $0.995^{30} = 0.860\,384$ (to 6 d.p.)

Percentage error = $\frac{0.860\,384 - 0.860\,368}{0.860\,384} \times 100 = 0.0019\%$

7 a $\left(3-\frac{x}{5}\right)^{10} = 3^{10} + \binom{10}{1}3^9\left(-\frac{x}{5}\right) + \binom{10}{2}3^8\left(-\frac{x}{5}\right)^2 + \ldots$
$= 59\,049 - 39\,366x + 11\,809.8x^2 + \ldots$

b We want $\left(3-\frac{x}{5}\right)^{10} = 2.98$
$\frac{x}{5} = 0.02$
$x = 0.1$

Substitute $x = 0.1$ into the expansion for $\left(3-\frac{x}{5}\right)^{10}$.

8 a $(1-3x)^5 = 1^5 + \binom{5}{1}1^4(-3x) + \binom{5}{2}1^3(-3x)^2 + \binom{5}{3}1^2(-3x)^3 + \ldots$
$= 1 - 15x + 90x^2 - 270x^3 + \ldots$

b For the expansion $(1 - 3x)^5$, only use the first two terms as x^2 and higher powers can be ignored.
$(1 + x)(1 - 3x)^5 \approx (1 + x)(1 - 15x)$
$\approx 1 - 15x + x - 15x^2$
$\approx 1 - 14x$

9 a So that higher powers of p can be ignored as they tend to 0.

b $(1-p)^{200} = 1^{200} + \binom{200}{1}1^{199}(-p) + \binom{200}{2}1^{198}(-p)^2 + \ldots$

$\approx 1 - 200p + 19\,900p^2$

c $1 - 200p + 19\,900p^2 = 0.92$

$19\,900p^2 - 200p + 0.08 = 0$

$$p = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(19\,900)(0.08)}}{2(19\,900)}$$

$$= \frac{200 \pm \sqrt{33\,632}}{39\,800}$$

$p = 0.009\,63$ or $p = 0.000\,417$ (to 3 s.f.)

As $p < 0.001$, the maximum value for p would be $0.000\,417$.