## The binomial expansion 8B

**1 a** $4! = 4 \times 3 \times 2 \times 1$
$= 24$

**b** $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 362\,880$

**c** $\dfrac{10!}{7!} = \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \dfrac{3\,628\,800}{5040}$ or $10 \times 9 \times 8$ since the 7! on numerator and denominator cancel

$= 720$

**d** $\dfrac{15!}{13!} = \dfrac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \dfrac{1\,307\,674\,368\,000}{6\,227\,020\,800}$ or $15 \times 14$ because the 13! cancels

$= 210$

**2 a** $\dbinom{4}{2} = \dfrac{4!}{2!2!}$

$= \dfrac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$

$= \dfrac{4 \times 3}{2 \times 1}$

$= 6$

**b** $\dbinom{6}{4} = \dfrac{6!}{4!2!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$

$= \dfrac{6 \times 5}{2 \times 1}$

$= 15$

**c** ${}^6C_3 = \dfrac{6!}{3!3!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1}$

$= \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1}$

$= 20$

**2 d** $\binom{5}{4} = \frac{5!}{4!1!}$

$= \frac{5\times4\times3\times2\times1}{4\times3\times2\times1\times1}$

$= \frac{5}{1}$

$= 5$

**e** $^{10}C_8 = \frac{10!}{8!2!}$

$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1\times2\times1}$

$= \frac{10\times9}{2\times1}$

$= 45$

**f** $\binom{9}{5} = \frac{9!}{5!4!}$

$= \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1\times4\times3\times2\times1}$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1}$

$= 126$

**3 a** $\binom{15}{6} = 5005$

**b** $^{10}C_7 = 120$

**c** $\binom{20}{10} = 184\,756$

**d** $\binom{20}{17} = 1140$

**e** $^{14}C_9 = 2002$

**f** $^{18}C_5 = 8568$

**4** The $r$th entry in the $n$th row of Pascal's triangle is given by $^{n-1}C_{r-1}$.

**a** $^{5-1}C_{2-1} = {}^4C_1$

**b** $^{6-1}C_{3-1} = {}^5C_2$

**c** $= {}^6C_2$

**d** $^{7-1}C_{4-1} = {}^6C_3$

**5** 5th number on the 12th row = ${}^{12-1}C_{5-1} = {}^{11}C_4 = 330$

**6 a** ${}^{11-1}C_{4-1} = {}^{10}C_3 = 120$

${}^{11-1}C_{5-1} = {}^{10}C_4 = 210$

**b** The coefficients are 1, 10, 45, 120, 210, ...
The term in $x^3$ is $120(1)^7(2x)^3 = 960x^3$.
Coefficient = 960

**7 a** ${}^{14-1}C_{4-1} = {}^{13}C_3 = 286$

${}^{14-1}C_{5-1} = {}^{13}C_4 = 715$

**b** The coefficients are 1, 13, 78, 286, 715, ...
The term in $x^4$ is $715(1)^9(3x)^4 = 57\,915x^4$.
Coefficient = 57 915

**8** $\binom{20}{10}0.5^{20} = {}^{20}C_{10}0.5^{20}$

$= 184\,756 \times 0.5^{20}$

$= 0.1762$ (to 4 s.f.)

Whilst this seems a low probability, there is more chance of the coin landing on 10 heads than any other number of heads.

**9 a** $^{n}C_1 = \frac{n!}{1!(n-1)!} = \frac{n\times(n-1)\times(n-2)\times\ldots\times 2\times 1}{1\times(n-1)\times(n-2)\times(n-3)\times\ldots\times 2\times 1} = n$

**b** $^{n}C_2 = \frac{n!}{2!(n-2)!} = \frac{n\times(n-1)\times(n-2)\times\ldots\times 2\times 1}{1\times 2\times(n-2)\times(n-3)\times\ldots\times 2\times 1} = \frac{n(n-1)}{2}$

**10** $\binom{50}{13} = \frac{50!}{13!a!}$

$\binom{50}{13} = \frac{50!}{13!37!}$

$a = 37$

**11** $\binom{35}{p} = \frac{35!}{p!18!}$

$\binom{35}{17} = \frac{35!}{17!18!}$

$p = 17$

**Challenge**

**a** $^{10}C_3 = \frac{10!}{3!7!} = 120$

$^{10}C_7 = \frac{10!}{7!3!} = 120$

**b** $^{14}C_5 = \frac{14!}{5!9!} = 2002$

$^{14}C_9 = \frac{14!}{9!5!} = 2002$

**c** The two answers for part **a** are the same and the two answers for part **b** are the same.

**d** $^{n}C_r = \frac{n!}{r!(n-r)!}$ and $^{n}C_{n-r} = \frac{n!}{(n-r)!r!}$ because $\frac{n!}{(n-r)!\big(n-(n-r)\big)!}$ and $(n-(n-r)) = r$

As $\frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!}$, $^{n}C_r = {}^{n}C_{n-r}$