

Straight line graphs, Mixed Exercise 5

1 a Gradient $m = -\frac{5}{12}$, $(x_1, y_1) = (2, 1)$

The equation of the line is:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{5}{12}(x - 2)$$

$$y - 1 = -\frac{5}{12}x + \frac{5}{6}$$

$$y = -\frac{5}{12}x + \frac{11}{6}$$

b Substitute $(k, 11)$ into $y = -\frac{5}{12}x + \frac{11}{6}$

$$11 = -\frac{5}{12}k + \frac{11}{6}$$

$$11 - \frac{11}{6} = -\frac{5}{12}k$$

$$\frac{55}{6} = -\frac{5}{12}k$$

Multiply each side by 12:

$$110 = 5k$$

$$k = -22$$

2 a The gradient of AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

So:

$$\frac{(2k-1)-1}{8-k} = \frac{1}{3}$$

$$\frac{2k-1-1}{8-k} = \frac{1}{3}$$

$$\frac{2k-2}{8-k} = \frac{1}{3}$$

Multiply each side by $(8 - k)$:

$$2k - 2 = \frac{1}{3}(8 - k)$$

Multiply each term by 3:

$$6k - 6 = 8 - k$$

$$7k - 6 = 8$$

$$7k = 14$$

$$k = 2$$

b $k = 2$.

So A and B have coordinates $(2, 1)$ and $(8, 3)$.

2 b The equation of the line is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{3 - 1} = \frac{x - 2}{8 - 2}$$

$$\frac{y - 1}{2} = \frac{x - 2}{6}$$

Multiply each side by 2:

$$y - 1 = \frac{1}{3}(x - 2)$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

3 a The equation of L_1 is:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{7}(x - 2)$$

$$y - 2 = \frac{1}{7}x - \frac{2}{7}$$

$$y = \frac{1}{7}x + \frac{12}{7}$$

The equation of L_2 is:

$$y - y_1 = (x - x_1)$$

$$y - 8 = -1(x - 4)$$

$$y - 8 = -x + 4$$

$$y = -x + 12$$

b Solve $y = \frac{1}{7}x + \frac{12}{7}$ and $y = -x + 12$ simultaneously.

$$-x + 12 = \frac{1}{7}x + \frac{12}{7}$$

$$12 = \frac{8}{7}x + \frac{12}{7}$$

$$10\frac{2}{7} = \frac{8}{7}x$$

$$x = \frac{10\frac{2}{7}}{\frac{8}{7}}$$

$$= 9$$

Substitute $x = 9$ into $y = -x + 12$:

$$y = -9 + 12$$

$$= 3$$

The lines intersect at $C(9, 3)$.

4 a The equation of l is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{6 - 0} = \frac{x - 1}{5 - 1}$$

$$\frac{y}{6} = \frac{x - 1}{4}$$

$$y = 6 \frac{(x - 1)}{4}$$

Multiply each side by 6:

$$y = \frac{3}{2}(x - 1)$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

b Solve $2x + 3y = 15$ and $y = \frac{3}{2}x - \frac{3}{2}$ simultaneously.

Substitute:

$$2x + 3\left(\frac{3}{2}x - \frac{3}{2}\right) = 15$$

$$2x + \frac{9}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x = \frac{39}{2}$$

$$13x = 39$$

$$x = 3$$

Substitute $x = 3$ into $y = \frac{3}{2}x - \frac{3}{2}$:

$$y = \frac{3}{2}(3) - \frac{3}{2}$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

The coordinates of C are $(3, 3)$.

5 $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (-19, -19)$

The equation of L is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-19 - 3} = \frac{x - 1}{-19 - 1}$$

$$\frac{y - 3}{-22} = \frac{x - 1}{-20}$$

$$\frac{y - 3}{-22} = \frac{x - 1}{-20}$$

5 Multiply each side by -22 :

$$y - 3 = \frac{-22}{-20}(x - 1)$$

$$y - 3 = \frac{11}{10}(x - 1)$$

Multiply each term by 10:

$$10y - 30 = 11(x - 1)$$

$$10y - 30 = 11x - 11$$

$$10y = 11x + 19$$

$$0 = 11x - 10y + 19$$

The equation of L is

$$11x - 10y + 19 = 0.$$

6 a $(x_1, y_1) = (2, 2)$, $(x_2, y_2) = (6, 0)$

The equation of l_1 is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{0 - 2} = \frac{x - 2}{6 - 2}$$

$$\frac{y - 2}{-2} = \frac{x - 2}{4}$$

$$\frac{y - 2}{-2} = \frac{x - 2}{4}$$

Multiply each side by -2 :

$$y - 2 = -\frac{1}{2}(x - 2) \quad (\text{Note: } -\frac{2}{4} = -\frac{1}{2})$$

$$y - 2 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 3$$

b The equation of l_2 is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{4}(x - (-9))$$

$$y = \frac{1}{4}(x + 9)$$

$$y = \frac{1}{4}x + \frac{9}{4}$$

7 $A(1, 3\sqrt{3})$, $B(2 + \sqrt{3}, 3 + 4\sqrt{3})$

The gradient of the line through A and B is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 + 4\sqrt{3} - 3\sqrt{3}}{2 + \sqrt{3} - 1}$$

$$= \frac{3 + \sqrt{3}}{1 + \sqrt{3}}$$

7 Rationalising the denominator:

$$\frac{(3 + \sqrt{3}) \times (1 - \sqrt{3})}{(1 + \sqrt{3}) \times (1 - \sqrt{3})} = \frac{3 - 3\sqrt{3} + \sqrt{3} - 3}{1 - 3}$$

$$= \frac{-2\sqrt{3}}{-2}$$

$$= \sqrt{3}$$

The equation of the line is:

$$y = \sqrt{3}x + c$$

Substituting $x = 1$ and $y = 3\sqrt{3}$ into

$$y = \sqrt{3}x + c:$$

$$3\sqrt{3} = \sqrt{3} + c$$

$$c = 2\sqrt{3}$$

The equation of line l is:

$$y = \sqrt{3}x + 2\sqrt{3}$$

Line l meets the x -axis when $y = 0$.

When $y = 0$, $x = -2$.

C is the point $(-2, 0)$.

8 a $A(-4, 6)$, $B(2, 8)$
The gradient of AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{2 - (-4)}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

The gradient of a line perpendicular to AB is:

$$-\frac{1}{\frac{1}{3}} = -3$$

The equation of p is:

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -3(x - 2)$$

$$y - 8 = -3x + 6$$

$$y = -3x + 14$$

b Substitute $x = 0$ in the equation for AB :

$$y = -3(0) + 14 = 14$$

The coordinates of C are $(0, 14)$.

9 a The line passes through $A(0, 4)$ and is perpendicular to l : $2x - y - 1 = 0$.

$$2x - y - 1 = 0$$

$$2x - 1 = y$$

$$y = 2x - 1$$

The gradient of $2x - y - 1 = 0$ is 2.

The gradient of a line perpendicular to $2x - y - 1 = 0$ is $-\frac{1}{2}$.

The equation of the line m is:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2}(x - 0)$$

$$y - 4 = -\frac{1}{2}x$$

$$y = -\frac{1}{2}x + 4$$

Or, since A is a y -intercept, the equation can be written once the gradient is known i.e $y = -(\frac{1}{2})x + 4$.

b To find P , solve $y = -\frac{1}{2}x + 4$ and $2x - y - 1 = 0$ simultaneously.

Substitute:

$$2x - (-\frac{1}{2}x + 4) - 1 = 0$$

$$2x + \frac{1}{2}x - 4 - 1 = 0$$

$$\frac{5}{2}x - 5 = 0$$

$$5x = 10$$

$$x = 2$$

Substitute $x = 2$ into $y = -\frac{1}{2}x + 4$:

$$y = -\frac{1}{2}(2) + 4$$

$$= -1 + 4$$

$$= 3$$

The lines intersect at $P(2, 3)$, as required.

c A line parallel to the line m has gradient $-\frac{1}{2}$.

The equation of the line n is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{2}(x - 3)$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

- 9 c To find Q solve $2x - y - 1 = 0$ and $y = -\frac{1}{2}x + \frac{3}{2}$ simultaneously.

Substitute:

$$2x - \left(-\frac{1}{2}x + \frac{3}{2}\right) - 1 = 0$$

$$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$$

$$\frac{5}{2}x - \frac{5}{2} = 0$$

$$\frac{5}{2}x = \frac{5}{2}$$

$$x = 1$$

Substitute $x = 1$ into $y = -\frac{1}{2}x + \frac{3}{2}$:

$$y = -\frac{1}{2}(1) + \frac{3}{2}$$

$$= -\frac{1}{2} + \frac{3}{2}$$

$$= 1$$

The lines intersect at $Q(1, 1)$.

- 10 $A(0, -2)$ and $B(6, 7)$

The gradient of the line through A and B is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{6 - 0}$$

$$= \frac{9}{6}$$

$$= \frac{3}{2}$$

The equation of the line through A and B is:

$$y = \frac{3}{2}x + c$$

Substituting $x = 0$ and $y = -2$ into

$$y = \frac{3}{2}x + c:$$

$$-2 = \frac{3}{2}(0) + c \text{ so } c = -2$$

As in Q9, the point A is the y -intercept so the equation can be written once the gradient has been calculated.

$$l_1: y = \frac{3}{2}x - 2$$

$$l_2: x + y = 8$$

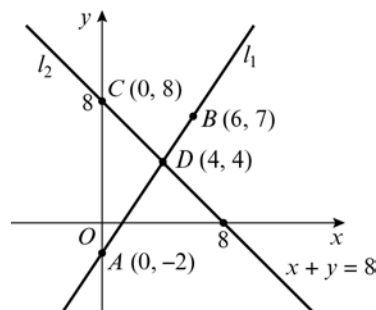
To find point D , solve simultaneously by substituting l_1 into l_2 .

$$x + \frac{3}{2}x - 2 = 8$$

$$\frac{5}{2}x = 10$$

$$x = 4$$

- 10 when $x = 4$,
 $4 + y = 8$,
 $y = 4$
 $\therefore D$ is the point $(4, 4)$.



The base of the triangle AC is 10 units.
 The height of the triangle is 4 units.
 Area $\triangle ACD$ is $\frac{1}{2} \times 10 \times 4 = 20$ units²

- 11 a $A(2, 16)$ and $B(12, -4)$

The equation of l_1 through A and B is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}$$

$$\frac{y - 16}{-20} = \frac{x - 2}{10}$$

Multiply each side by -20 :

$$y - 16 = -2(x - 2) \left(\text{Note: } -\frac{20}{10} = -2 \right)$$

$$y - 16 = -2x + 4$$

$$y = -2x + 20$$

$$2x + y = 20$$

- b The equation of l_2 through $C(-1, 1)$ with gradient $\frac{1}{3}$ is:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3}(x - (-1))$$

$$y - 1 = \frac{1}{3}(x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

12 a $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$

The gradient of AB is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - (-2)}{7 - (-1)} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

b Since ABC is a right angle the gradient of BC is:

$$\frac{-1}{\frac{1}{2}} = -2$$

$$\text{So } \frac{y_2 - y_1}{x_2 - x_1} = -2$$

$$\Rightarrow \frac{4 - 2}{k - 7} = -2$$

$$\Rightarrow \frac{2}{k - 7} = -2$$

Multiply each side by $(k - 7)$:

$$2 = -2(k - 7)$$

$$2 = -2k + 14$$

$$2k = 12$$

$$k = 6$$

c The equation of the line passing through B and C is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{4 - 2} = \frac{x - 7}{6 - 7}$$

$$\frac{y - 2}{2} = \frac{x - 7}{-1}$$

Multiply each side by 2:

$$y - 2 = -2(x - 7)$$

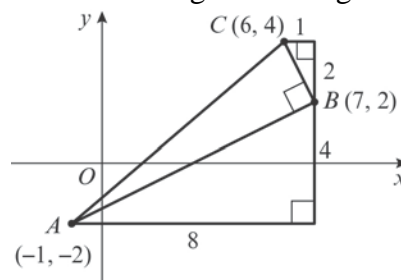
$$y - 2 = -2x + 14$$

$$y = -2x + 16$$

$$2x + y = 16$$

$$2x + y - 16 = 0$$

d Remember angle B is a right angle.



Use the diagram or the distance formula to find the lengths AB and BC .

$$AB = \sqrt{8^2 + 4^2}$$

$$= \sqrt{80}$$

$$BC = \sqrt{1^2 + 2^2}$$

$$= \sqrt{5}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times \sqrt{80} \times \sqrt{5}$$

$$= \frac{1}{2} \times \sqrt{400}$$

$$= \frac{1}{2} \times 20$$

$$= 10 \text{ units}^2$$

13 a The equation of the line through $(-1, 5)$ and $(4, -2)$ is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-2 - 5} = \frac{x - (-1)}{4 - (-1)}$$

$$\frac{y - 5}{-7} = \frac{x + 1}{5}$$

Multiply each side by -35 :

$$5(y - 5) = -7(x + 1)$$

$$5y - 25 = -7x - 7$$

$$\text{(Note: } \frac{-35}{-7} = 5 \text{ and } \frac{-35}{5} = -7)$$

$$7x + 5y - 25 = -7$$

$$7x + 5y - 18 = 0$$

13 b For the coordinates of A , substitute

$$y = 0:$$

$$7x + 5(0) - 18 = 0$$

$$7x - 18 = 0$$

$$7x = 18$$

$$x = \frac{18}{7}$$

The coordinates of A are $(\frac{18}{7}, 0)$.

For the coordinates of B , substitute

$$x = 0:$$

$$7(0) + 5y - 18 = 0$$

$$5y - 18 = 0$$

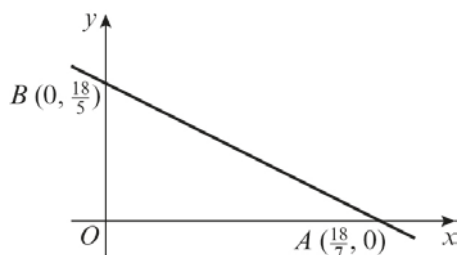
$$5y = 18$$

$$y = \frac{18}{5}$$

The coordinates of B are $(0, \frac{18}{5})$.

The area of $\triangle OAB$ is:

$$\frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{162}{35}$$



14 a Rearrange $l_1: 4y + x = 0$ into the form

$$y = mx + c:$$

$$4y = -x$$

$$y = -\frac{1}{4}x$$

l_1 has gradient $-\frac{1}{4}$ and it meets the coordinate axes at $(0, 0)$.

l_2 has gradient 2 and it meets the y -axis at $(0, -3)$.

l_2 meets the x -axis when $y = 0$.

Substitute $y = 0$ into the equation:

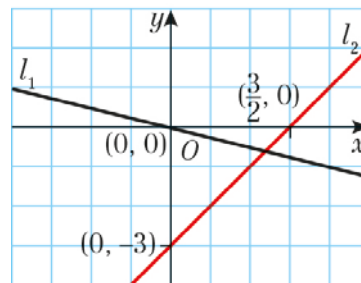
$$0 = 2x - 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

l_2 meets the x -axis at $(\frac{3}{2}, 0)$.

14 a



b Solve $4y + x = 0$ and $y = 2x - 3$ simultaneously.

Substitute:

$$4(2x - 3) + x = 0$$

$$8x - 12 + x = 0$$

$$9x - 12 = 0$$

$$9x = 12$$

$$x = \frac{12}{9}$$

$$x = \frac{4}{3}$$

Now substitute $x = \frac{4}{3}$ into $y = 2x - 3$:

$$y = 2\left(\frac{4}{3}\right) - 3$$

$$= \frac{8}{3} - 3$$

$$= -\frac{1}{3}$$

The coordinates of A are $(\frac{4}{3}, -\frac{1}{3})$.

c The gradient of l_1 is $-\frac{1}{4}$.

The gradient of a line perpendicular to

$$l_1 \text{ is } -\frac{1}{-\frac{1}{4}} = 4.$$

The equation of this line is:

$$y - y_1 = m(x - x_1)$$

$$y - \left(-\frac{1}{3}\right) = 4\left(x - \frac{4}{3}\right)$$

$$y + \frac{1}{3} = 4x - \frac{16}{3}$$

$$y = 4x - \frac{17}{3}$$

Multiply each term by 3:

$$3y = 12x - 17$$

$$0 = 12x - 3y - 17$$

The equation of the line is

$$12x - 3y - 17 = 0.$$

15 a $A(4, 6)$ and $B(12, 2)$

The gradient of the line l_1 through A and B is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - 6}{12 - 4} \\ &= \frac{-4}{8} \\ &= -\frac{1}{2} \end{aligned}$$

The equation of l_1 is:

$$y = -\frac{1}{2}x + c$$

Substituting $x = 4$ and $y = 6$ into

$$y = -\frac{1}{2}x + c:$$

$$6 = -\frac{1}{2}(4) + c$$

$$c = 8$$

$$y = -\frac{1}{2}x + c$$

$$x + 2y - 16 = 0$$

b The gradient of the line l_2 is $-\frac{2}{3}$, the y -intercept is 0 .

$$y = -\frac{2}{3}x$$

c Solve $x + 2y - 16 = 0$ and $y = -\frac{2}{3}x$ simultaneously.

$$x + 2(-\frac{2}{3}x) - 16 = 0$$

$$x - \frac{4}{3}x - 16 = 0$$

$$-\frac{1}{3}x = 16$$

$$x = -48$$

When $x = -48$:

$$y = -\frac{2}{3}(-48)$$

$$y = 32$$

C is the point $(-48, 32)$.

d The gradient of OA is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{6 - 0}{4 - 0} \\ &= \frac{3}{2} \end{aligned}$$

d The gradient of OC is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{32 - 0}{(-48) - 0} = -\frac{2}{3} \\ \frac{3}{2} \times -\frac{2}{3} &= -1. \end{aligned}$$

Therefore the lines OA and OC are perpendicular.

e $OA = \sqrt{(4-0)^2 + (6-0)^2}$

$$\begin{aligned} &= \sqrt{52} \\ &= \sqrt{4 \times 13} \\ &= 2\sqrt{13} \\ &= \sqrt{((-48)-0)^2 + (32-0)^2} \\ &= \sqrt{3328} \\ &= \sqrt{256 \times 13} \\ &= 16\sqrt{13} \end{aligned}$$

f Area of $\triangle OAB = \frac{1}{2} \times 16\sqrt{13} \times 2\sqrt{13}$

$$= 208 \text{ units}^2$$

16 a $(4a, a)$ and $(-3a, 2a)$

The distance d between the points is:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{((-3a) - 4a)^2 + (2a - a)^2} \\ &= \sqrt{49a^2 + a^2} \\ &= \sqrt{50a^2} \\ &= \sqrt{25 \times 2a^2} \\ &= 5a\sqrt{2} \end{aligned}$$

b For points $(4, 1)$ and $(-3, 2)$, $a = 1$.
Substitute $a = 1$ into $5a\sqrt{2}$.
Distance = $5\sqrt{2}$

c For points $(12, 3)$ and $(-9, 6)$, $a = 3$.
Substitute $a = 3$ into $5a\sqrt{2}$.
Distance = $15\sqrt{2}$

16 d For points $(-20, -5)$ and $(15, -10)$,
 $a = -5$.

Substitute $a = -5$ into $5a\sqrt{2}$.

$$\text{Distance} = -25\sqrt{2}$$

17 a (x, y) is a point on $y = 3x$, so its
 coordinates are $(x, 3x)$.
 The distance between $A(-1, 5)$ and $(x,$
 $3x)$ is:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x - (-1))^2 + (3x - 5)^2} \\ &= \sqrt{x^2 + 2x + 1 + 9x^2 - 30x + 25} \\ &= \sqrt{10x^2 - 28x + 26} \end{aligned}$$

b $\sqrt{10x^2 - 28x + 26} = \sqrt{74}$

$$10x^2 - 28x + 26 = 74$$

$$10x^2 - 28x - 48 = 0$$

$$5x^2 - 14x - 24 = 0$$

$$(5x + 6)(x - 4) = 0$$

$$x = -\frac{6}{5} \text{ or } x = 4$$

When $x = -\frac{6}{5}$, $y = 3(-\frac{6}{5}) = -\frac{18}{5}$

When $x = 4$, $y = 3(4) = 12$

The points are $B(-\frac{6}{5}, -\frac{18}{5})$ and

$C(4, 12)$.

c The gradient of the line $y = 3x$ is 3,
 so the perpendicular line has
 gradient $-\frac{1}{3}$.

Its equation is:

$$y = -\frac{1}{3}x + c$$

When $x = -1$ and $y = 5$:

$$5 = -\frac{1}{3}(-1) + c$$

$$c = \frac{14}{3}$$

$$y = -\frac{1}{3}x + \frac{14}{3}$$

d Solving $y = -\frac{1}{3}x + \frac{14}{3}$ and $y = 3x$
 simultaneously:

$$3x = -\frac{1}{3}x + \frac{14}{3}$$

17 d $9x = -x + 14$

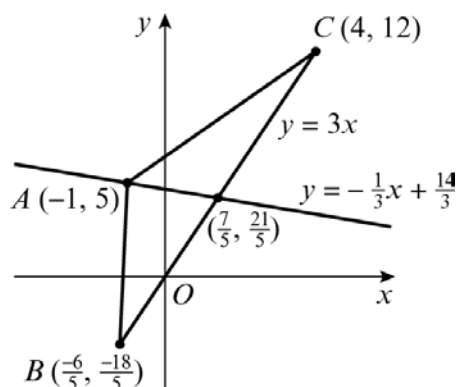
$$10x = 14$$

$$x = \frac{7}{5}$$

When $x = \frac{7}{5}$: $y = 3(\frac{7}{5}) = \frac{21}{5}$

The point is $(\frac{7}{5}, \frac{21}{5})$

e



$$\begin{aligned} BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-\frac{6}{5}))^2 + (12 - (-\frac{18}{5}))^2} \\ &= \sqrt{(\frac{26}{5})^2 + (\frac{78}{5})^2} \\ &= \sqrt{\frac{6760}{25}} \end{aligned}$$

Distance from $A(-1, 5)$ to $(\frac{7}{5}, \frac{21}{5})$ is:

$$\begin{aligned} &\sqrt{(\frac{7}{5} - (-1))^2 + (\frac{21}{5} - 5)^2} \\ &= \sqrt{(\frac{12}{5})^2 + (-\frac{4}{5})^2} \\ &= \sqrt{\frac{160}{25}} \end{aligned}$$

Area of triangle is:

$$\begin{aligned} \frac{1}{2} \times \sqrt{\frac{6760}{25}} \times \sqrt{\frac{160}{25}} &= \frac{520}{25} \\ &= 20.8 \text{ units}^2 \end{aligned}$$

18 a Gradient of the line of best fit is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{30\,000 - 22\,500}{3900 - 3200} \\ &= \frac{7500}{700} \\ &= 10.7 \end{aligned}$$

18 b $C = aP + b$

$a = 10.7$

Using the point (3200, 22 500):

$22\,500 = 10.7(3200) + b$

$b = -11\,740$

$C = 10.7P - 11\,740$

c a is the gradient, which is the increase in carbon dioxide emissions in millions of tonnes for every 1 million tonnes of oil pollution.

d The model is not valid for small values of P as a negative amount of carbon dioxide emissions is not possible.

Challenge

1 $(-2, -2)$, $B(13, 8)$ and $C(-4, 14)$

The equation of AB is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-2)}{8 - (-2)} = \frac{x - (-2)}{13 - (-2)}$$

$$\frac{y + 2}{10} = \frac{x + 2}{15}$$

$$3y + 6 = 2x + 4$$

$$3y = 2x - 2$$

$$y = \frac{2}{3}x - \frac{2}{3}$$

The gradient of $AB = \frac{2}{3}$.

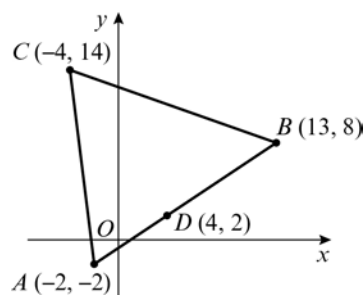
The gradient of a line perpendicular to $AB = -\frac{3}{2}$.

The equation of the perpendicular to AB through $C(-4, 14)$ is:

$$y - 14 = -\frac{3}{2}(x - (-4))$$

$$y - 14 = -\frac{3}{2}x - 6$$

$$y = -\frac{3}{2}x + 8$$



Point D is where the line and the perpendicular intersect.

Solve the equations $y = \frac{2}{3}x - \frac{2}{3}$ and

$y = -\frac{3}{2}x + 8$ simultaneously.

$$\frac{2}{3}x - \frac{2}{3} = -\frac{3}{2}x + 8$$

Multiply each term by 6.

$$4x - 4 = -9x + 48$$

$$13x = 52$$

$$x = 4$$

Now substitute $x = 4$ into

$$y = -\frac{3}{2}x + 8:$$

$$y = -\frac{3}{2}(4) + 8$$

$$y = 2$$

D is the point $(4, 2)$.

Challenge

$$\begin{aligned}
 1 \quad AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(13 - (-2))^2 + (8 - (-2))^2} \\
 &= \sqrt{15^2 + 10^2} \\
 &= \sqrt{325} \\
 CD &= \sqrt{(4 - (-4))^2 + (2 - 14)^2} \\
 &= \sqrt{8^2 + (-12)^2} \\
 &= \sqrt{208} \\
 \text{Area of } \triangle ABC &= \frac{1}{2} \times \sqrt{325} \times \sqrt{208} \\
 &= 130 \text{ units}^2
 \end{aligned}$$

2 $A(3, 8)$, $B(9, 9)$ and $C(5, 2)$

The gradient of AB is:

$$\begin{aligned}
 \frac{y_2 - y_1}{x_2 - x_1} &= \frac{9 - 8}{9 - 3} \\
 &= \frac{1}{6}
 \end{aligned}$$

l_1 is perpendicular to AB , so its gradient is -6 . It passes through C , so its equation is:

$$\begin{aligned}
 y &= -6x + c \\
 2 &= -6(5) + c \\
 c &= 32
 \end{aligned}$$

The equation of l_1 is $y = -6x + 32$.

The gradient of BC is:

$$\begin{aligned}
 \frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - 9}{5 - 9} \\
 &= \frac{7}{4}
 \end{aligned}$$

l_2 is perpendicular to BC , so its gradient is $-\frac{4}{7}$. It passes through A , so its equation is:

$$\begin{aligned}
 y &= -\frac{4}{7}x + c \\
 8 &= -\frac{4}{7}(3) + c \\
 c &= \frac{68}{7}
 \end{aligned}$$

The equation of l_2 is $y = -\frac{4}{7}x + \frac{68}{7}$.

2 The gradient of AC is:

$$\begin{aligned}
 \frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - 8}{5 - 3} \\
 &= -\frac{6}{2} \\
 &= -3
 \end{aligned}$$

l_3 is perpendicular to BC , so its gradient is $\frac{1}{3}$. It passes through B , so its equation is:

$$\begin{aligned}
 y &= \frac{1}{3}x + c \\
 9 &= \frac{1}{3}(9) + c \\
 c &= 6
 \end{aligned}$$

The equation of l_3 is $y = \frac{1}{3}x + 6$.

Solve l_1 and l_2 simultaneously.

$$\begin{aligned}
 -6x + 32 &= -\frac{4}{7}x + \frac{68}{7} \\
 -42x + 224 &= -4x + 68 \\
 38x &= 156 \\
 x &= \frac{78}{19} \\
 y &= -6\left(\frac{78}{19}\right) + 32 = \frac{140}{19}
 \end{aligned}$$

Their point of intersection is $\left(\frac{78}{19}, \frac{140}{19}\right)$.

Now solve l_2 and l_3 simultaneously.

$$\begin{aligned}
 -\frac{4}{7}x + \frac{68}{7} &= \frac{1}{3}x + 6 \\
 -12x + 204 &= 7x + 126 \\
 19x &= 78 \\
 x &= \frac{78}{19}
 \end{aligned}$$

$$y = \frac{1}{3}\left(\frac{78}{19}\right) + 6 = \frac{140}{19}$$

Their point of intersection is $\left(\frac{78}{19}, \frac{140}{19}\right)$.

Therefore, l_1 , l_2 and l_3 all intersect at $\left(\frac{78}{19}, \frac{140}{19}\right)$.

3 $A(0, 0)$, $B(a, b)$ and $C(c, 0)$

The gradient of AB is:

$$\begin{aligned}
 \frac{y_2 - y_1}{x_2 - x_1} &= \frac{b - 0}{a - 0} \\
 &= \frac{b}{a}
 \end{aligned}$$

l_1 is perpendicular to AB so its gradient is $-\frac{a}{b}$.

3 It passes through C so its equation is:

$$y = -\frac{a}{b}x + k \text{ where } k \text{ is the } y\text{-intercept.}$$

At C , $x = c$ and $y = 0$.

$$0 = -\frac{ac}{b} + k$$

$$k = \frac{ac}{b}$$

The equation of line l_1 is:

$$y = -\frac{a}{b}x + \frac{ac}{b}$$

The gradient of BC is:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{0 - b}{c - a} \\ &= \frac{-b}{c - a} \end{aligned}$$

l_2 is perpendicular to BC so its gradient is

$$\frac{c - a}{b}.$$

It passes through A , so its equation is:

$$y = \frac{c - a}{b}x + K \text{ where } K \text{ is the } y\text{-intercept.}$$

At A , $x = 0$, $y = 0$.

$$0 = \frac{c - a}{b}(0) + K$$

$$K = 0$$

The equation of line l_2 is $y = \frac{c - a}{b}x$.

l_3 is the vertical line through (a, b) , so its equation is $x = a$.

Solve l_1 and l_3 simultaneously.

$$\begin{aligned} y &= -\frac{a^2}{b} + \frac{ac}{b} \\ &= \frac{a(c - a)}{b} \end{aligned}$$

The intersection of l_1 and l_3 is the point

$$\left(a, \frac{a(c - a)}{b}\right).$$

Now solve l_2 and l_3 simultaneously.

$$y = \frac{a(c - a)}{b}$$

The intersection of l_2 and l_3 is the point

$$\left(a, \frac{a(c - a)}{b}\right).$$

Therefore, l_1 , l_2 and l_3 all intersect at

$$\left(a, \frac{a(c - a)}{b}\right).$$