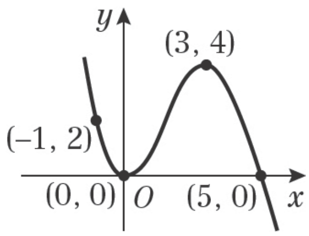
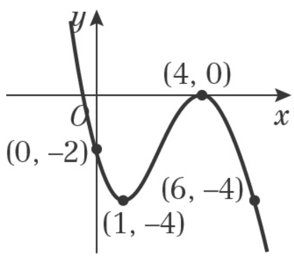
Graphs and transformations 4G

1 a f(x + 1) is a translation by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, or one unit to the left.



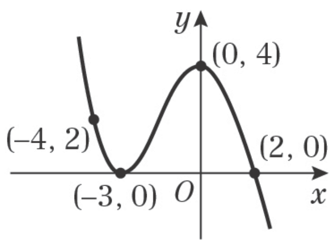
$A'(-1, 2)$, $B'(0, 0)$, $C'(3, 4)$, $D'(5, 0)$

b f(x) − 4 is a translation by $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, or four units down.



$A'(0, -2)$, $B'(1, -4)$, $C'(4, 0)$, $D'(6, -4)$

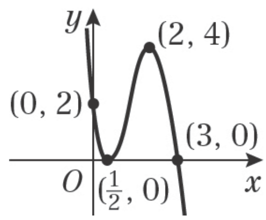
c f(x + 4) is a translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$, or four units to the left.



$A'(-4, 2)$, $B'(-3, 0)$, $C'(0, 4)$, $D'(2, 0)$

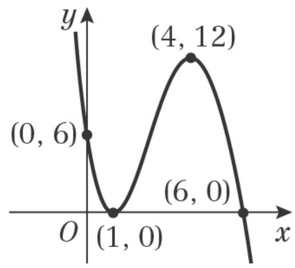
d f(2x) is a stretch with scale factor $\frac{1}{2}$ in the x-direction.

d



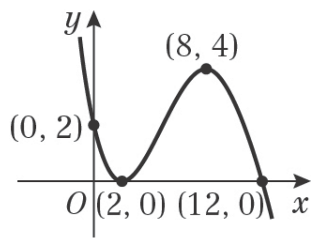
$A'(0, 2)$, $B'\left(\frac{1}{2}, 0\right)$, $C'(2, 4)$, $D'(3, 0)$

e 3f(x) is a stretch with scale factor 3 in the y-direction.



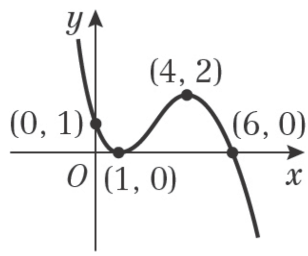
$A'(0, 6)$, $B'(1, 0)$, $C'(4, 12)$, $D'(6, 0)$

f $\text{f}\left(\frac{1}{2}x\right)$ is a stretch with scale factor 2 in the x-direction.



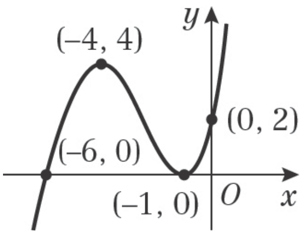
$A'(0, 2)$, $B'(2, 0)$, $C'(8, 4)$, $D'(12, 0)$

g $\frac{1}{2}$f(x) is a stretch with scale factor $\frac{1}{2}$ in the y-direction.



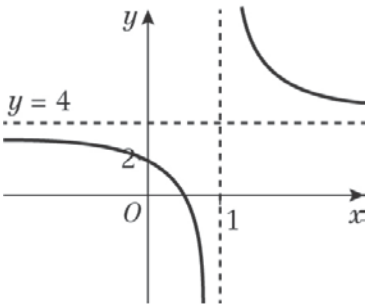
$A'(0, 1)$, $B'(1, 0)$, $C'(4, 2)$, $D'(6, 0)$

1 h f(−x) is a reflection in the y-axis



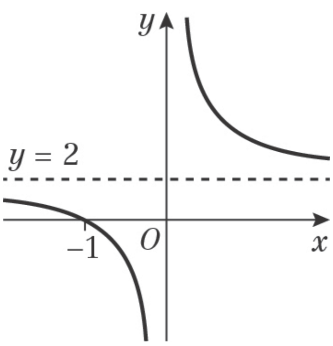
$A'(0, 2)$, $B'(-1, 0)$, $C'(-4, 4)$, $D'(-6, 0)$

2 a f(x) + 2 is a translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or two units up.



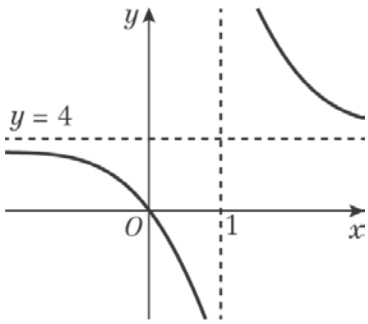
The curve crosses the x-axis at (0, 2) and the y-axis at (a, 0), where $0 < a < 1$.
The horizontal asymptote is $y = 4$.
The vertical asymptote is $x = 1$.

b f(x + 1) is a translation by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, or one unit to the left.



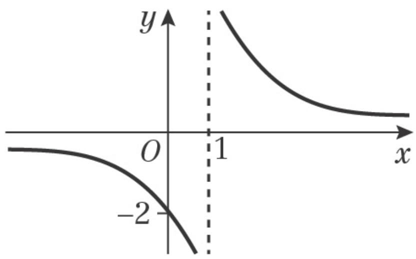
The curve crosses the x-axis at (−1, 0).
The horizontal asymptote is $y = 2$.
The vertical asymptote is $x = 0$.

2 c 2f(x) is a stretch with scale factor 2 in the y-direction.



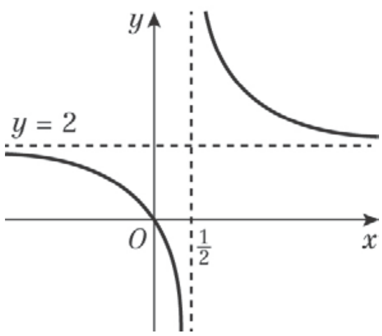
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = 4$.
The vertical asymptote is $x = 1$.

d f(x) − 2 is a translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, or two units down.



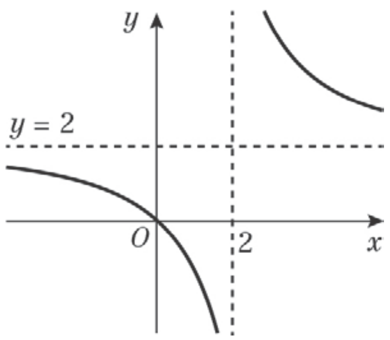
The curve crosses the y-axis at (0, −2).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = 1$.

e f(2x) is a stretch with scale factor $\frac{1}{2}$ in the x-direction.



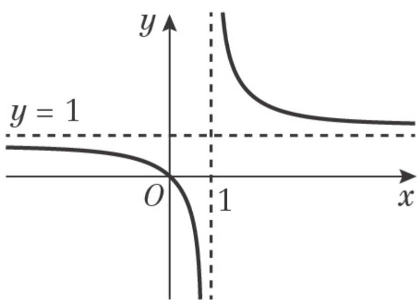
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = 2$.
The vertical asymptote is $x = \frac{1}{2}$.

2 f $f\left(\frac{1}{2}x\right)$ is a stretch with scale factor 2 in the x-direction.



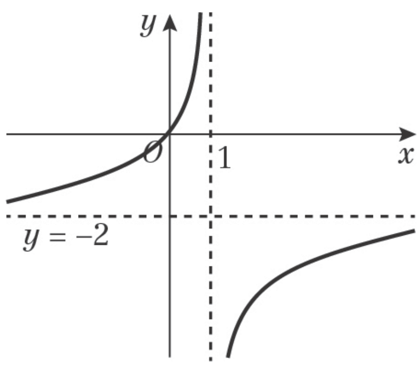
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = 2$.
The vertical asymptote is $x = 2$.

g $\frac{1}{2}f(x)$ is a stretch with scale factor $\frac{1}{2}$ in the y-direction.



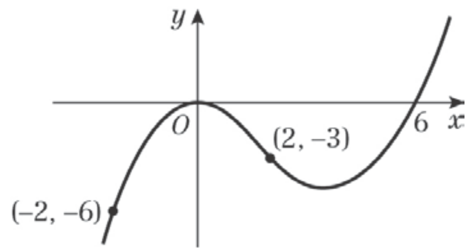
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = 1$.
The vertical asymptote is $x = 1$.

h $-f(x)$ is a reflection in the x-axis.



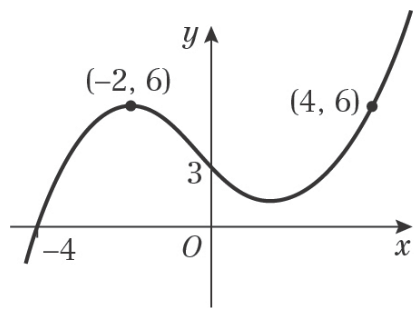
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = -2$.
The vertical asymptote is $x = 1$.

3 a $f(x-2)$ is a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, or two units to the right.



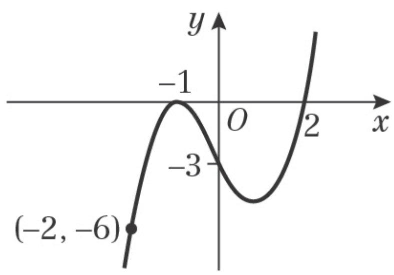
$A'(-2, -6)$, $B'(0, 0)$, $C'(2, -3)$, $D'(6, 0)$

b $f(x) + 6$ is a translation by $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$, or six units up.



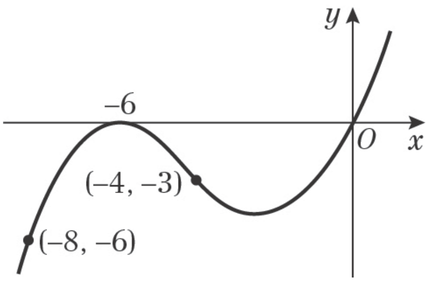
$A'(-4, 0)$, $B'(-2, 6)$, $C'(0, 3)$, $D'(4, 6)$

c $f(2x)$ is a stretch with scale factor $\frac{1}{2}$ in the x-direction.



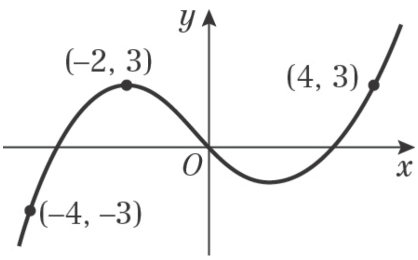
$A'(-2, -6)$, $B'(-1, 0)$, $C'(0, -3)$, $D'(2, 0)$

3 d f(x + 4) is a translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$, or four units to the left.



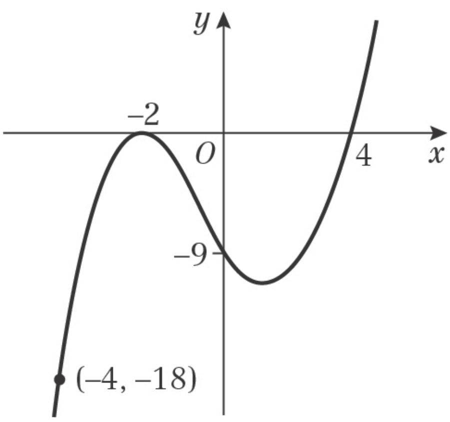
$A'(-8, -6)$, $B'(-6, 0)$, $C'(-4, -3)$, $D'(0, 0)$

e f(x) + 3 is a translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$, or three units up.



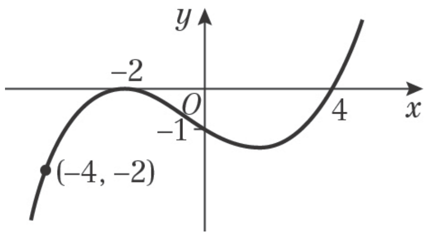
$A'(-4, -3)$, $B'(-2, 3)$, $C'(0, 0)$, $D'(4, 3)$

f 3f(x) is a stretch with scale factor 3 in the y-direction.



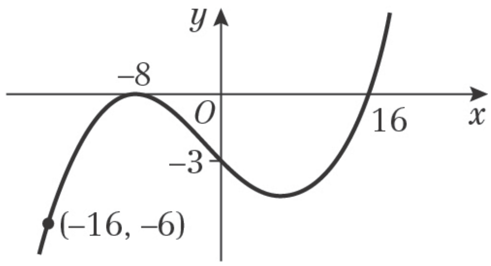
$A'(-4, -18)$, $B'(-2, 0)$, $C'(0, -9)$, $D'(4, 0)$

g $\frac{1}{3}$f(x) is a stretch with scale factor $\frac{1}{3}$ in the y-direction.



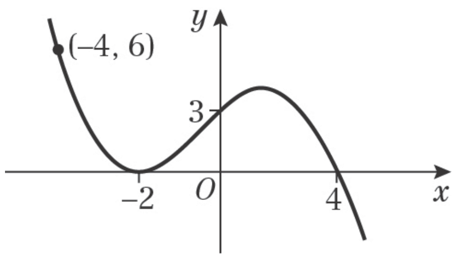
$A'(-4, -2)$, $B'(-2, 0)$, $C'(0, -1)$, $D'(4, 0)$

h $\mathrm{f}\left(\frac{1}{4}x\right)$ is a stretch with scale factor 4 in the x-direction.



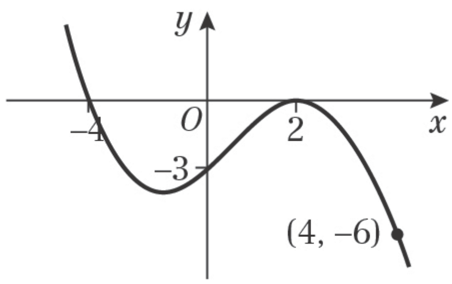
$A'(-16, -6)$, $B'(-8, 0)$, $C'(0, -3)$, $D'(16, 0)$

i −f(x) is a reflection in the x-axis.



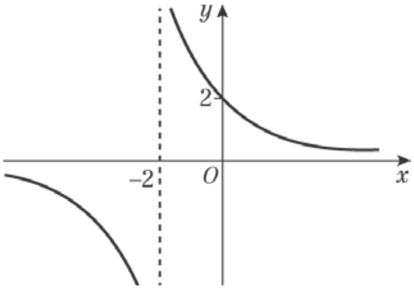
$A'(-4, 6)$, $B'(-2, 0)$, $C'(0, 3)$, $D'(4, 0)$

j f(−x) is a reflection in the y-axis.



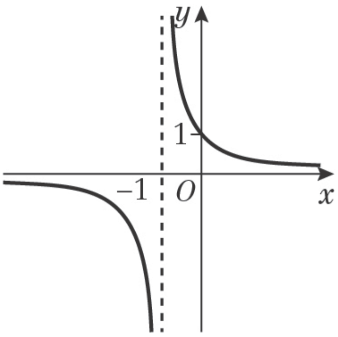
$A'(4, -6)$, $B'(2, 0)$, $C'(0, -3)$, $D'(-4, 0)$

4 a i $2f(x)$ is a stretch with scale factor 2 in the y-direction.



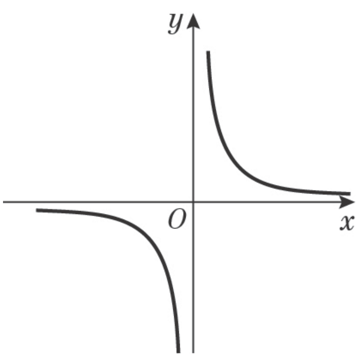
The curve crosses the y-axis at (0, 2).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = -2$.

ii $f(2x)$ is a stretch with scale factor $\frac{1}{2}$ in the x-direction.



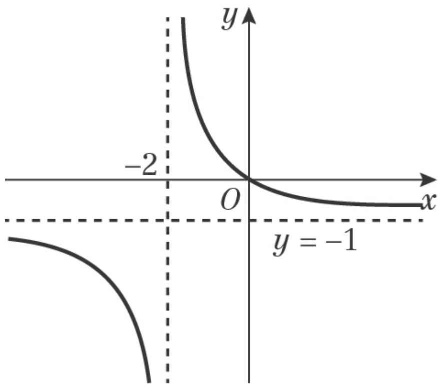
The curve crosses the y-axis at (0, 1).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = -1$.

iii $f(x - 2)$ is a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, or two units to the right.



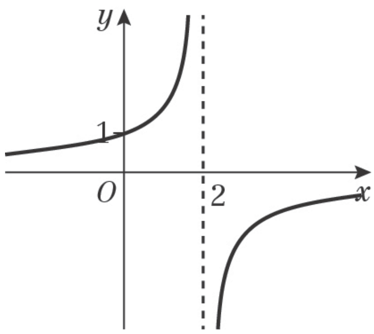
There are no intersections with the axes.
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = 0$.

iv $f(x) - 1$ is a translation by $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, or one unit down.



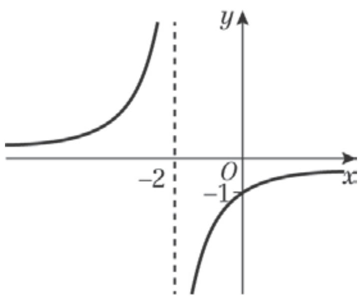
The curve crosses the axes at (0, 0).
The horizontal asymptote is $y = -1$.
The vertical asymptote is $x = -2$.

v $f(-x)$ is a reflection in the y-axis.



The curve crosses the y-axis at (0, 1).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = 2$.

vi $-f(x)$ is a reflection in the x-axis.



The curve crosses the y-axis at (0, −1).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = -2$.

4 b The shape of the curve is like $y = \frac{k}{x}$, $k > 0$.

$x = -2$ asymptote suggests denominator is zero when $x = -2$, so denominator is $x + 2$. Also, $f(0) = 1$ means the numerator must be 2.

$f(x) = \frac{2}{x+2}$

5 a $P(2, 1)$ is mapped to $Q(4, 1)$.
The x-coordinate has doubled, which is a stretch with scale factor 2 in the x-direction.

$y = f\left(\frac{1}{2}x\right)$

$a = \frac{1}{2}$

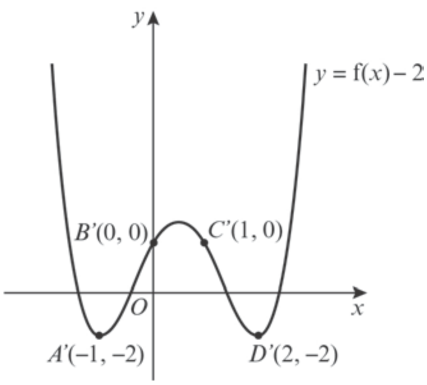
b i $f(x - 4)$ is a translation by $\begin{pmatrix}4\\0\end{pmatrix}$, or four units to the right.
So P is mapped to $(6, 1)$.

ii $3f(x)$ is a stretch with scale factor 3 in the y-direction.
So P is mapped to $(2, 3)$.

iii $\frac{1}{2}f(x) - 4$ is a stretch with scale factor $\frac{1}{2}$ in the y-direction and then a translation by $\begin{pmatrix}0\\-4\end{pmatrix}$, or four units down.

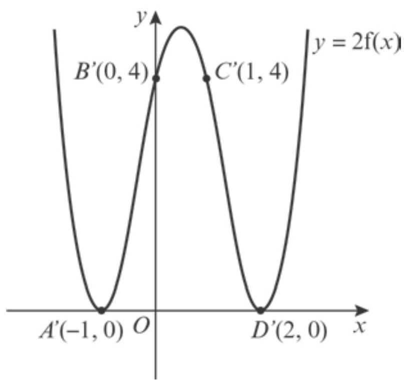
So P is mapped to $\left(2, -3\frac{1}{2}\right)$

6 a $y + 2 = f(x)$
$y = f(x) - 2$, which is a translation by $\begin{pmatrix}0\\-2\end{pmatrix}$, or two units down.



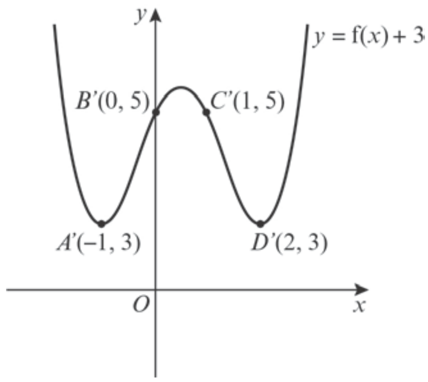
$A'(-1, -2)$, $B'(0, 0)$, $C'(1, 0)$, $D'(2, -2)$

6 b $\frac{1}{2}y = f(x)$
$y = 2f(x)$, which is a stretch with scale factor 2 in the y-direction.



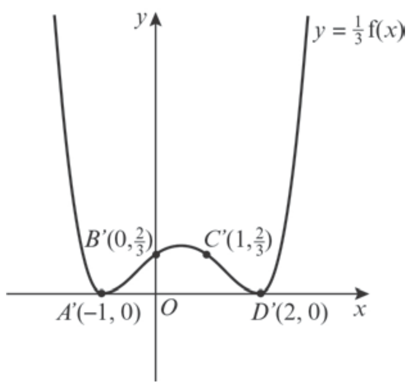
$A'(-1, 0)$, $B'(0, 4)$, $C'(1, 4)$, $D'(2, 0)$

c $y - 3 = f(x)$
$y = f(x) + 3$, which is a translation by $\begin{pmatrix}0\\3\end{pmatrix}$, or three units up.



$A'(-1, 3)$, $B'(0, 5)$, $C'(1, 5)$, $D'(2, 3)$

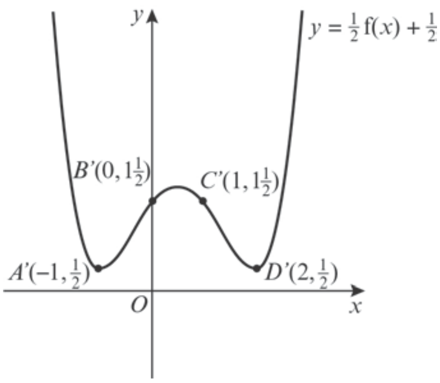
d $3y = f(x)$
$y = \frac{1}{3}f(x)$, which is a stretch with scale factor $\frac{1}{3}$ in the y-direction.



$A'(-1, 0)$, $B'\left(0, \frac{2}{3}\right)$, $C'\left(1, \frac{2}{3}\right)$, $D'(2, 0)$

6 e $2y - 1 = f(x)$

$y = \frac{1}{2}f(x) + \frac{1}{2}$, which is a stretch with scale factor $\frac{1}{2}$ in the y-direction, then a translation by $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$, or $\frac{1}{2}$ unit up.



$A'\left(-1, \frac{1}{2}\right)$, $B'\left(0, 1\frac{1}{2}\right)$, $C'\left(1, 1\frac{1}{2}\right)$,
$D'\left(2, \frac{1}{2}\right)$