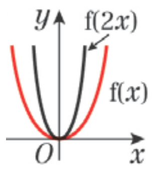
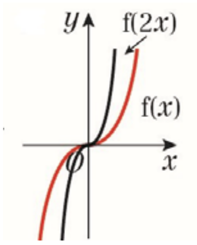
Graphs and transformations 4F

1 a f(2x) is a stretch with scale factor $\frac{1}{2}$ in the x-direction.

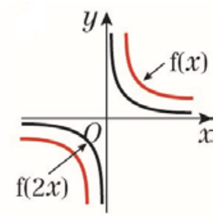
i $f(x) = x^2$, $f(2x) = (2x)^2 = 4x^2$



ii $f(x) = x^3$, $f(2x) = (2x)^3 = 8x^3$

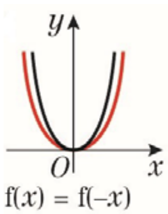


iii $f(x) = \dfrac{1}{x}$, $f(2x) = \dfrac{1}{2x} = \frac{1}{2} \times \dfrac{1}{x}$

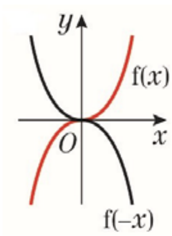


b f(−x) is a reflection in the y-axis (or stretch with scale factor −1 in the x-direction).

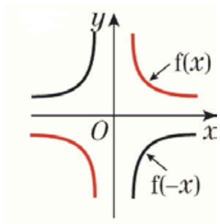
i $f(x) = x^2$, $f(-x) = (-x)^2 = x^2$



ii $f(x) = x^3$, $f(-x) = (-x)^3 = -x^3$

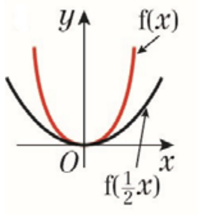


b iii $f(x) = \dfrac{1}{x}$, $f(-x) = \dfrac{1}{-x} = -\dfrac{1}{x}$

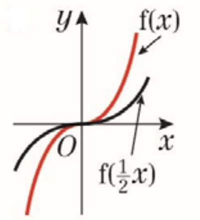


c $f\left(\frac{1}{2}x\right)$ is a stretch with scale factor 2 in the x-direction.

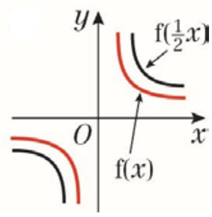
i $f(x) = x^2$, $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \dfrac{x^2}{4}$



ii $f(x) = x^3$, $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^3 = \dfrac{x^3}{8}$

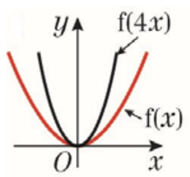


c iii $f(x) = \dfrac{1}{x}$, $f\left(\frac{1}{2}x\right) = \dfrac{1}{\frac{1}{2}x} = \dfrac{2}{x}$

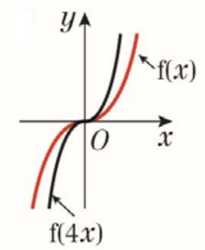


d f(4x) is a stretch with scale factor $\frac{1}{4}$ in the x-direction.

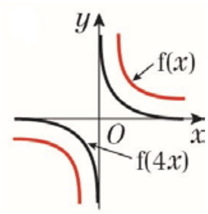
i $f(x) = x^2$, $f(4x) = (4x)^2 = 16x^2$



1 d ii $f(x) = x^3$, $f(4x) = (4x)^3 = 64x^3$

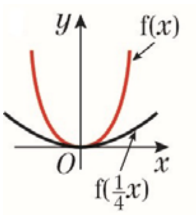


iii $f(x) = \frac{1}{x}$, $f(4x) = \frac{1}{4x} = \frac{1}{4} \times \frac{1}{x}$

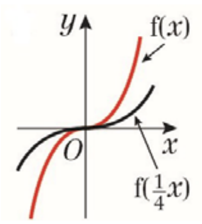


e $f\left(\frac{1}{4}x\right)$ is a stretch with scale factor 4 in the x-direction.

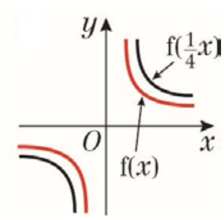
i $f(x) = x^2$, $f\left(\frac{1}{4}x\right) = \left(\frac{1}{4}x\right)^2 = \frac{x^2}{16}$



ii $f(x) = x^3$, $f\left(\frac{1}{4}x\right) = \left(\frac{1}{4}x\right)^3 = \frac{x^3}{64}$

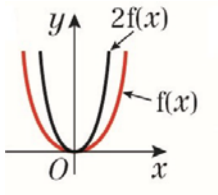


e iii $f(x) = \frac{1}{4}$, $f\left(\frac{1}{4}x\right) = \frac{1}{\frac{1}{4}x} = \frac{4}{x}$

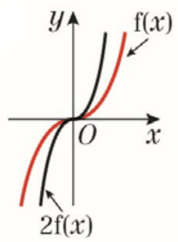


f $2f(x)$ is a stretch with scale factor 2 in the y-direction.

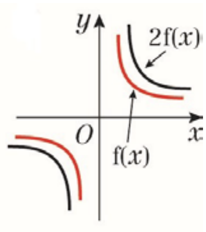
i $f(x) = x^2$, $2f(x) = 2x^2$



ii $f(x) = x^3$, $2f(x) = 2x^3$

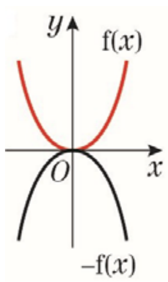


iii $f(x) = \frac{1}{x}$, $2f(x) = 2 \times \frac{1}{x} = \frac{2}{x}$

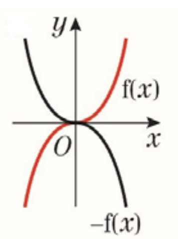


g $-f(x)$ is a reflection in the x-axis (or stretch with scale factor -1 in the y-direction).

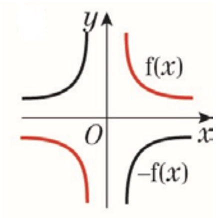
i $f(x) = x^2$, $-f(x) = -x^2$



ii $f(x) = x^3$, $-f(x) = -x^3$

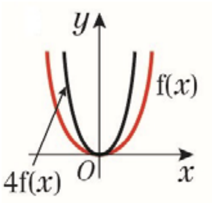


1 g iii $f(x) = \frac{1}{x}$, $-f(x) = -\frac{1}{x}$

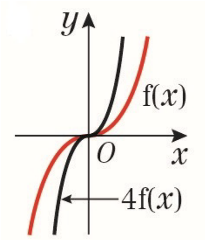


h 4f(x) is a stretch with scale factor 4 in the y-direction.

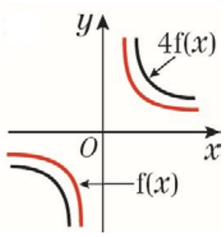
i $f(x) = x^2$, $4f(x) \to y = 4x^2$



ii $f(x) = x^3$, $4f(x) = 4x^3$

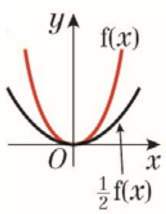


iii $f(x) = \frac{1}{x}$, $4f(x) = 4 \times \frac{1}{x} = \frac{4}{x}$

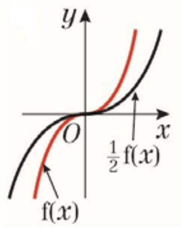


i i $\frac{1}{2}f(x)$ is a stretch with scale factor $\frac{1}{2}$ in the y-direction.

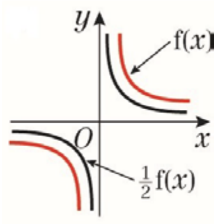
$f(x) = x^2$, $\frac{1}{2}f(x) = \frac{1}{2}x^2$



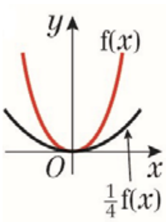
i ii $f(x) = x^3$, $\frac{1}{2}f(x) = \frac{1}{2}x^3$



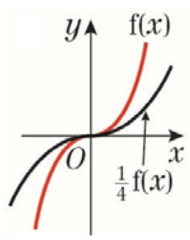
iii $f(x) = \frac{1}{x}$, $\frac{1}{2}f(x) = \frac{1}{2} \times \frac{1}{x} = \frac{1}{2x}$



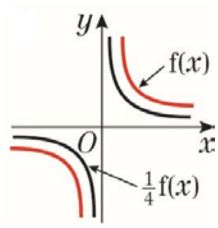
j i $\frac{1}{4}f(x)$ is a stretch with scale factor $\frac{1}{4}$ in the y-direction.
$f(x) = x^2$, $\frac{1}{4}f(x) = \frac{1}{4}x^2$



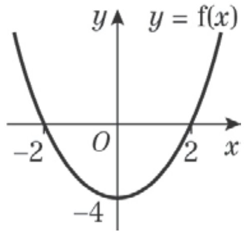
ii $f(x) = x^3$, $\frac{1}{4}f(x) = \frac{1}{4}x^3$



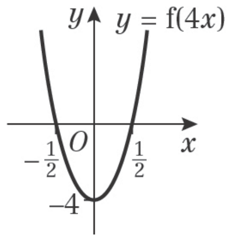
iii $f(x) = \frac{1}{x}$, $\frac{1}{4}f(x) = \frac{1}{4} \times \frac{1}{x} = \frac{1}{4x}$



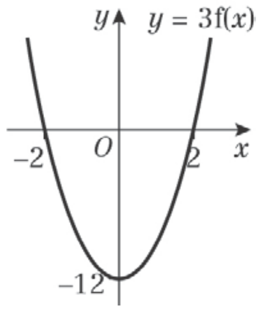
2 a $y = x^2 - 4$
$= (x-2)(x+2)$
As $a = 1$ is positive, the graph has a ∪ shape and a minimum point.
$0 = (x-2)(x+2)$
So $x = 2$ or $x = -2$
The curve crosses the x-axis at (2, 0) and (−2, 0).
When $x = 0$, $y = (-2) \times 2 = -4$
The curve crosses the y-axis at (0, −4).



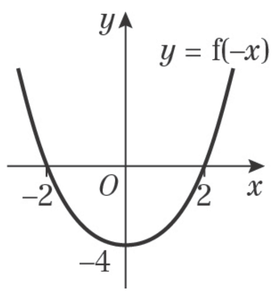
b f(4x) is a stretch with scale factor $\frac{1}{4}$ in the x-direction.



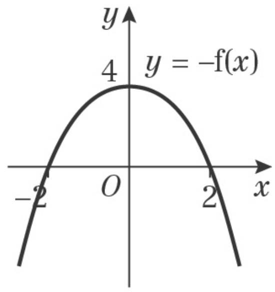
$\frac{1}{3}y = f(x)$
$y = 3f(x)$
3f(x) is a stretch with scale factor 3 in the y-direction.



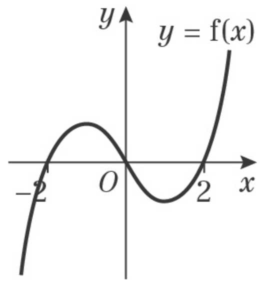
f(−x) is a reflection in the y-axis.



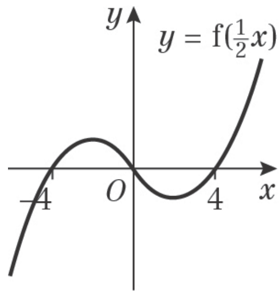
2 b −f(x) is a reflection in the x-axis.



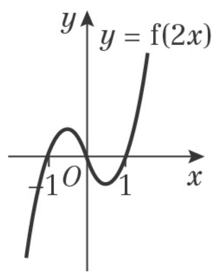
3 a $y = (x-2)(x+2)x$
$0 = (x-2)(x+2)x$
So $x = 2$, $x = -2$ or $x = 0$
The curve crosses the x-axis at (2, 0), (−2, 0) and (0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



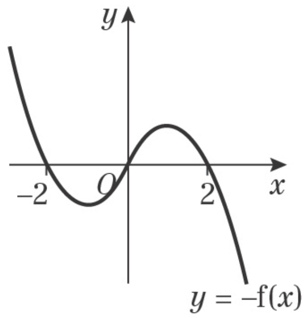
b $f\left(\frac{1}{2}x\right)$ is a stretch with scale factor 2 in the x-direction.



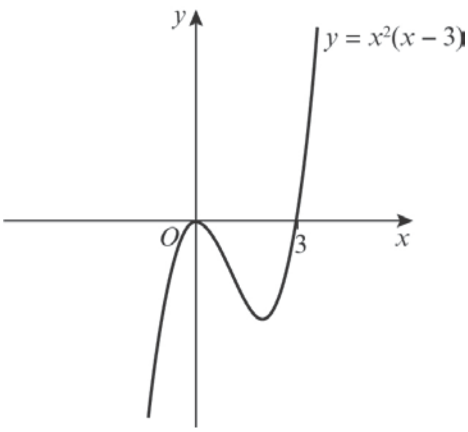
f(2x) is a stretch with scale factor $\frac{1}{2}$ in the x-direction.



3 b $-\text{f}(x)$ is a reflection in the x-axis.

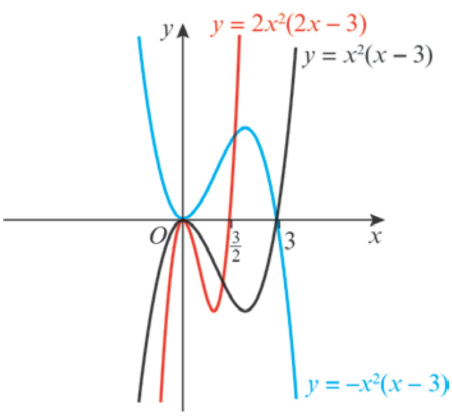


4 a $y = x^2(x - 3)$
$0 = x^2(x - 3)$
So $x = 0$ or $x = 3$
The curve touches the x-axis at (0, 0) and crosses it at (3, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

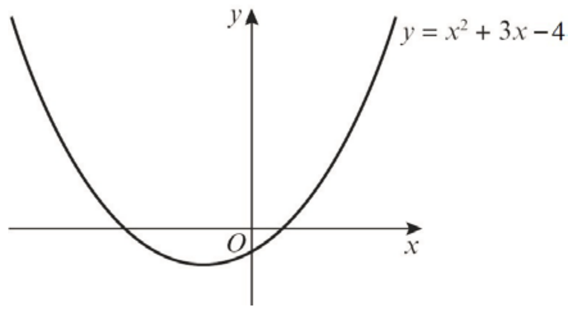


b i $\text{f}(x) = x^2(x - 3)$, so $y = (2x)^2(2x - 3)$ is $\text{f}(2x)$, which is a stretch with scale factor $\frac{1}{2}$ in the x-direction.

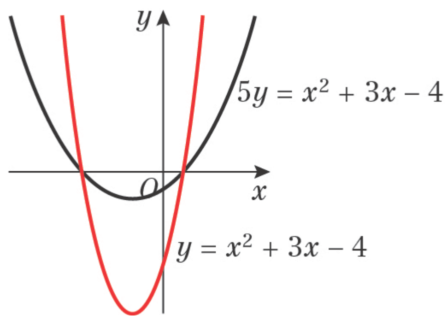
ii $y = -x^2(x - 3)$ is $-\text{f}(x)$, which is a reflection in the x-axis.



5 a $y = x^2 + 3x - 4$
$= (x + 4)(x - 1)$
As $a = 1$ is positive, the graph has a ∪ shape and a minimum point.
$0 = (x + 4)(x - 1)$
So $x = -4$ or $x = 1$
The curve crosses the x-axis at (−4, 0) and (1, 0).
When $x = 0$, $y = 4 \times (-1) = -4$
The curve crosses the y-axis at (0, −4).

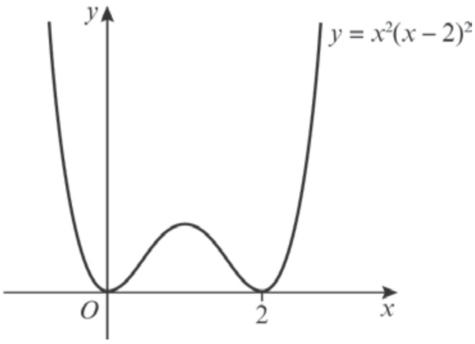


b $5y = x^2 + 3x - 4$
$y = \frac{1}{5}(x^2 + 3x - 4)$
$\text{f}(x) = x^2 + 3x - 4$, so $y = \frac{1}{5}(x^2 + 3x - 4)$ is $\frac{1}{5}\text{f}(x)$, which is a stretch with scale factor $\frac{1}{5}$ in the y-direction.

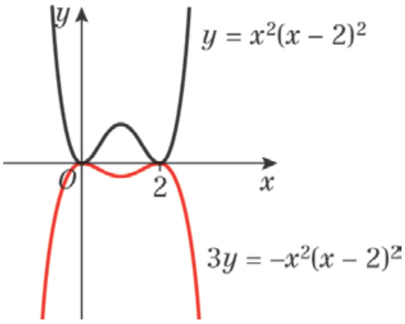


6 a $y = x^2(x - 2)^2$
$0 = x^2(x - 2)^2$
So $x = 0$ or $x = 2$
The curve touches the x-axis at (0, 0) and (2, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

6 a



b $3y = -x^2(x-2)^2$
$y = -\frac{1}{3}x^2(x-2)^2$
$f(x) = x^2(x-2)^2$, so $y = -\frac{1}{3}x^2(x-2)^2$ is
$\left(\frac{1}{2}x\right)-\frac{1}{3}f(x)$, which is a stretch with scale factor $\frac{1}{3}$ in the y-direction and a reflection in the x-axis.



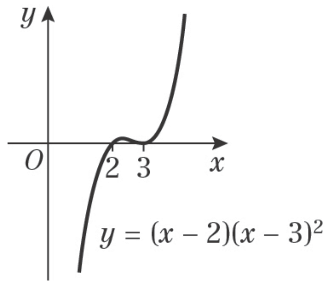
7 a $y = f(2x)$ is a stretch with scale factor $\frac{1}{2}$ in the x-direction, so all x-coordinates are halved.
$P(2, -3)$ is transformed to the point $(1, -3)$.

b $y = 4f(x)$ is a stretch with scale factor 4 in the y-direction, so all y-coordinates are multiplied by four.
$P(2, -3)$ is transformed to the point $(2, -12)$.

8 $f(\frac{1}{2}x)$ is a stretch with scale factor 2 in the x-direction, so all x-coordinates are doubled.
$Q(-2, 8)$ is transformed to the point $(-4, 8)$.

9 a $y = (x-2)(x-3)^2$
$0 = (x-2)(x-3)^2$
So $x = 2$ or $x = 3$
The curve crosses the x-axis at $(2, 0)$ and touches it at $(3, 0)$.
When $x = 0$, $y = (-2) \times (-3)^2 = -18$

$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $f(x) = (x-2)(x-3)^2$
$y = (ax-2)(ax-3)^2$ is the graph of $y = f(ax)$, which is a stretch with scale factor $\frac{1}{a}$ in the x-direction, so all x-coordinates are multiplied by $\frac{1}{a}$.
For the coordinate $(2, 0)$ to be transformed to $(1, 0)$, multiply the x-coordinate by $\frac{1}{2}$, giving $a = 2$.
For the coordinate $(3, 0)$ to be transformed to $(1, 0)$, multiply the x-coordinate by $\frac{1}{3}$, giving $a = 3$, $a = 2$ or $a = 3$

Challenge

1 $y = \frac{1}{3}f(2x)$ is a stretch with scale factor $\frac{1}{3}$ in the y-direction, so multiply the y-coordinate by $\frac{1}{3}$ and a stretch with scale factor $\frac{1}{2}$ in the x-direction, so multiply the x-coordinate by $\frac{1}{2}$.
$R(4, -6)$ is transformed to $(2, -2)$.

2 $S(-4, 7)$ is transformed to $S'(-8, 1.75)$.
The x-coordinate has doubled, which is a stretch of scale factor 2 in the x-direction.
The y-coordinate has been divided by 4, which is a stretch of scale factor $\frac{1}{4}$ in the y-direction.
The transformation is $y = \frac{1}{4}f\left(\frac{1}{2}x\right)$.