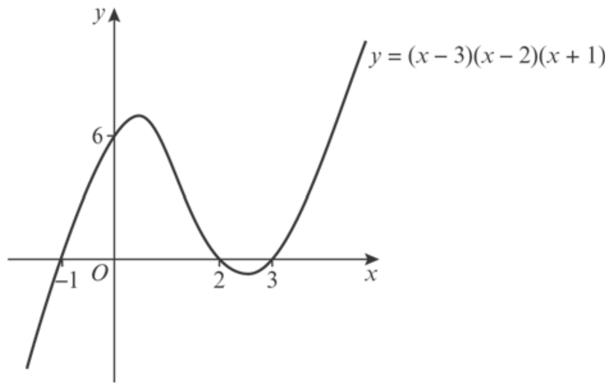
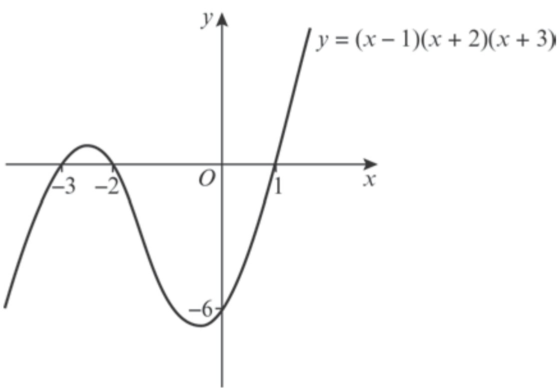
Graphs and transformations 4A

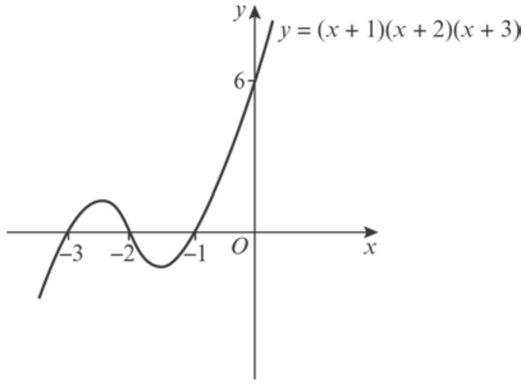
1 a $y = (x - 3)(x - 2)(x + 1)$
$0 = (x - 3)(x - 2)(x + 1)$
So $x = 3$, $x = 2$ or $x = -1$
So the curve crosses the x-axis at $(3, 0)$, $(2, 0)$ and $(-1, 0)$.
When $x = 0$, $y = (-3) \times (-2) \times 1 = 6$
So the curve crosses the y-axis at $(0, 6)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



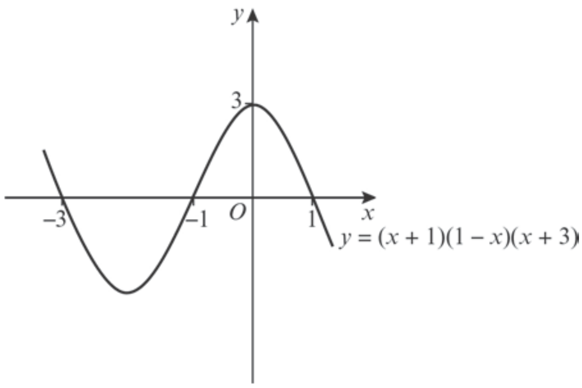
b $y = (x - 1)(x + 2)(x + 3)$
$0 = (x - 1)(x + 2)(x + 3)$
So $x = 1$, $x = -2$ or $x = -3$
So the curve crosses the x-axis at $(1, 0)$, $(-2, 0)$ and $(-3, 0)$.
When $x = 0$, $y = (-1) \times 2 \times 3 = -6$
So the curve crosses the y-axis at $(0, -6)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



c $y = (x + 1)(x + 2)(x + 3)$
$0 = (x + 1)(x + 2)(x + 3)$
So $x = -1$, $x = -2$ or $x = -3$
So the curve crosses the x-axis at $(-1, 0)$, $(-2, 0)$ and $(-3, 0)$.
When $x = 0$, $y = 1 \times 2 \times 3 = 6$
So the curve crosses the y-axis at $(0, 6)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



d $y = (x + 1)(1 - x)(x + 3)$
$0 = (x + 1)(1 - x)(x + 3)$
So $x = -1$, $x = 1$ or $x = -3$
So the curve crosses the x-axis at $(-1, 0)$, $(1, 0)$ and $(-3, 0)$.
When $x = 0$, $y = 1 \times 1 \times 3 = 3$
So the curve crosses the y-axis at $(0, 3)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



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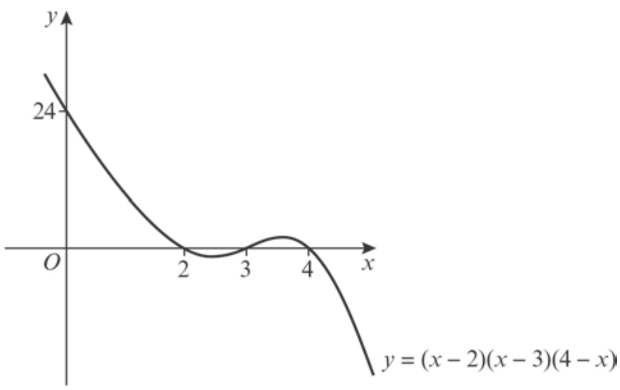
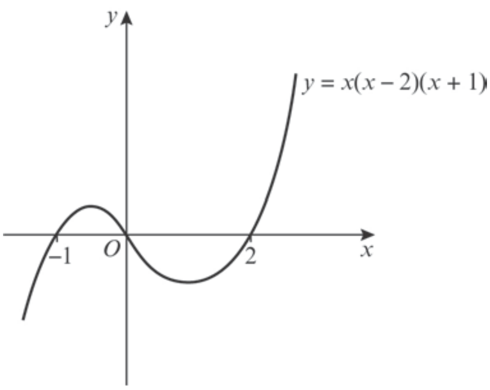
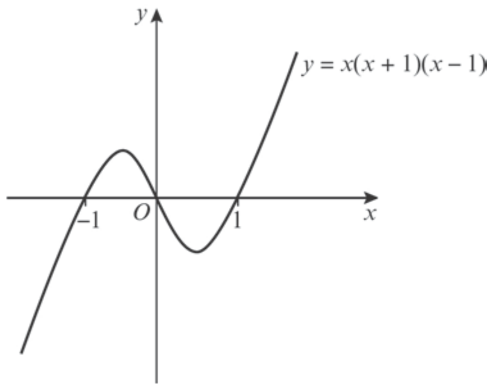
1 e $y = (x - 2)(x - 3)(4 - x)$
$0 = (x - 2)(x - 3)(4 - x)$
So $x = 2$, $x = 3$ or $x = 4$
So the curve crosses the x-axis at
(2, 0), (4, 0) and (4, 0).
When $x = 0$, $y = (-2) \times (-3) \times 4 = 24$
So the curve crosses the y-axis at (0, 24).
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



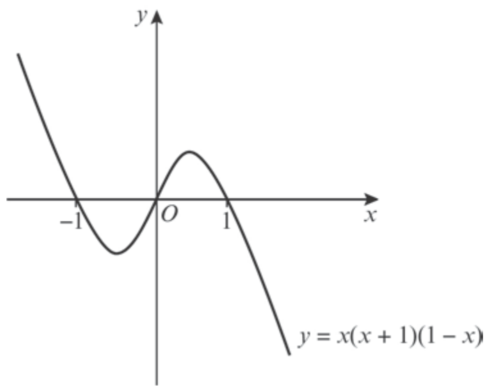
f $y = x(x - 2)(x + 1)$
$0 = x(x - 2)(x + 1)$
So $x = 0$, $x = 2$ or $x = -1$
So the curve crosses the x-axis at
(0, 0), (2, 0) and (−1, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



g $y = x(x + 1)(x - 1)$
$0 = x(x + 1)(x - 1)$
So $x = 0$, $x = -1$ or $x = 1$
So the curve crosses the x-axis at
(0, 0), (−1, 0) and (1, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



h $y = x(x + 1)(1 - x)$
$0 = x(x + 1)(1 - x)$
So $x = 0$, $x = -1$ or $x = 1$
So the curve crosses the x-axis at
(0, 0), (−1, 0) and (1, 0).
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



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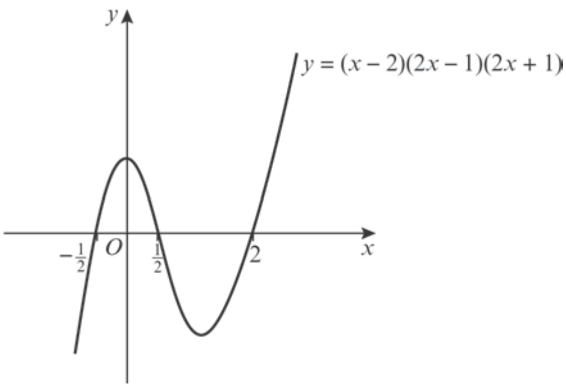
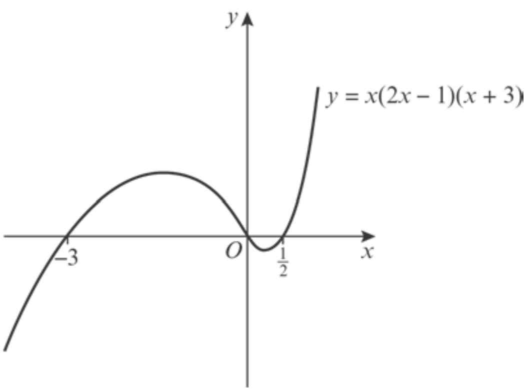
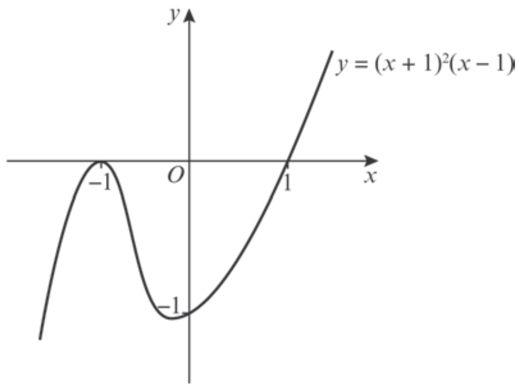
1 i $y = (x - 2)(2x - 1)(2x + 1)$
$0 = (x - 2)(2x - 1)(2x + 1)$
So $x = 2$, $x = \frac{1}{2}$ or $x = -\frac{1}{2}$
So the curve crosses the x-axis at
$(2, 0)$, $\left(\frac{1}{2}, 0\right)$ and $\left(-\frac{1}{2}, 0\right)$.
When $x = 0$, $y = (-2) \times (-1) \times 1 = 2$
So the curve crosses the y-axis at $(0, 2)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



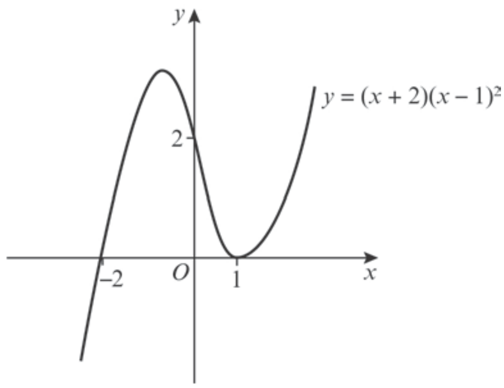
j $y = x(2x - 1)(x + 3)$
$0 = x(2x - 1)(x + 3)$
So $x = 0$, $x = \frac{1}{2}$ or $x = -3$
So the curve crosses the x-axis at
$(0, 0)$, $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



2 a $y = (x + 1)^2(x - 1)$
$0 = (x + 1)^2(x - 1)$
So $x = -1$ or $x = 1$
So the curve crosses the x-axis at $(1, 0)$
and touches the x-axis at $(-1, 0)$.
When $x = 0$, $y = 1^2 \times (-1) = -1$
So the curve crosses the y-axis at $(0, -1)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $y = (x + 2)(x - 1)^2$
$0 = (x + 2)(x - 1)^2$
So $x = -2$ or $x = 1$
So the curve crosses the x-axis at $(-2, 0)$
and touches the x-axis at $(1, 0)$.
When $x = 0$, $y = 2 \times (-1)^2 = 2$
So the curve crosses the y-axis at $(0, 2)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



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2 c $y = (2 - x)(x + 1)^2$

$0 = (2 - x)(x + 1)^2$

So $x = 2$ or $x = -1$

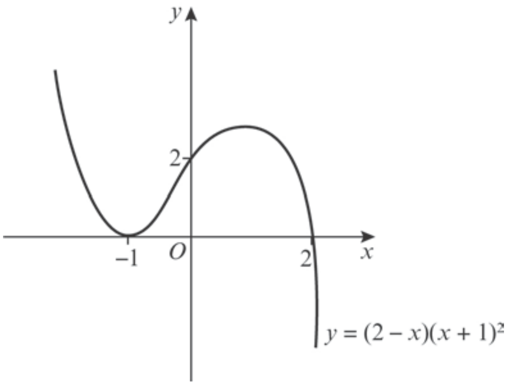
So the curve crosses the x-axis at (2, 0) and touches the x-axis at (−1, 0).

When $x = 0$, $y = 2 \times 1^2 = 2$

So the curve crosses the y-axis at (0, 2).

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



d $y = (x - 2)(x + 1)^2$

$0 = (x - 2)(x + 1)^2$

So $x = 2$ or $x = -1$

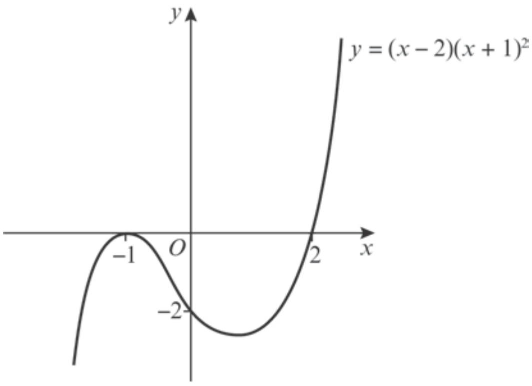
So the curve crosses the x-axis at (2, 0) and touches the x-axis at (−1, 0).

When $x = 0$, $y = (-2) \times 1^2 = -2$

So the curve crosses the y-axis at (0, −2).

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



e $y = x^2(x + 2)$

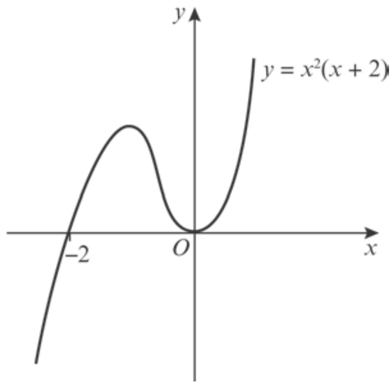
$0 = x^2(x + 2)$

So $x = 0$ or $x = -2$

So the curve crosses the x-axis at (−2, 0) and touches the x-axis at (0, 0).

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



f $y = (x - 1)^2x$

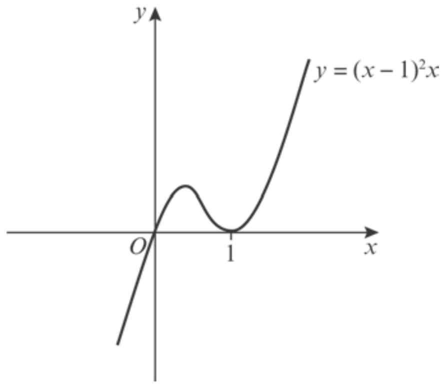
$0 = (x - 1)^2x$

So $x = 1$ or $x = 0$

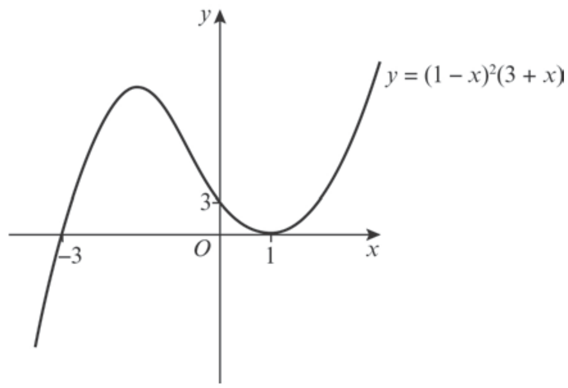
So the curve crosses the x-axis at (0, 0) and touches the x-axis at (1, 0).

$x \to \infty, y \to \infty$

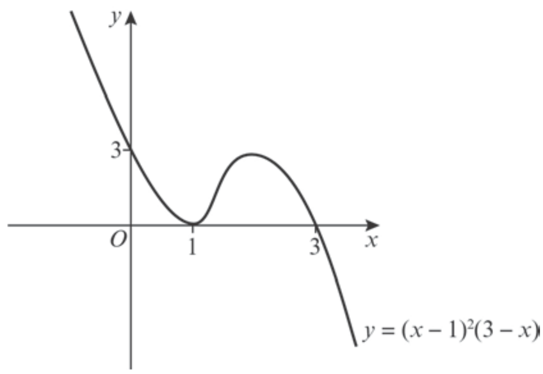
$x \to -\infty, y \to -\infty$



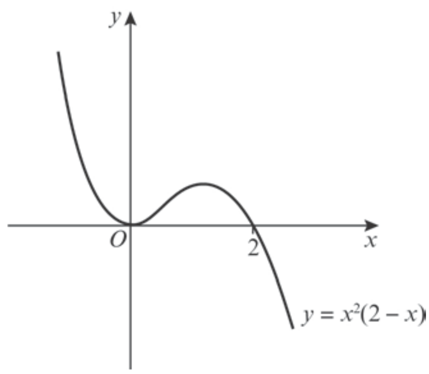
2 g $y = (1 - x)^2(3 + x)$
$0 = (1 - x)^2(3 + x)$
So $x = 1$ or $x = -3$
So the curve crosses the x-axis at $(-3, 0)$ and touches the x-axis at $(1, 0)$.
When $x = 0$, $y = 1^2 \times 3 = 3$
So the curve crosses the y-axis at $(0, 3)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



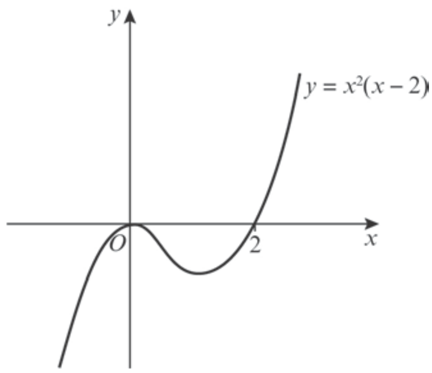
h $y = (x - 1)^2(3 - x)$
$0 = (x - 1)^2(3 - x)$
So $x = 1$ or $x = 3$
So the curve crosses the x-axis at $(3, 0)$ and touches the x-axis at $(1, 0)$.
When $x = 0$, $y = (-1)^2 \times 3 = 3$
So the curve crosses the y-axis at $(0, 3)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



i $y = x^2(2 - x)$
$0 = x^2(2 - x)$
So $x = 0$ or $x = 2$
So the curve crosses the x-axis at $(2, 0)$ and touches the x-axis at $(0, 0)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



j $y = x^2(x - 2)$
$0 = x^2(x - 2)$
So $x = 0$ or $x = 2$
So the curve crosses the x-axis at $(2, 0)$ and touches the x-axis at $(0, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



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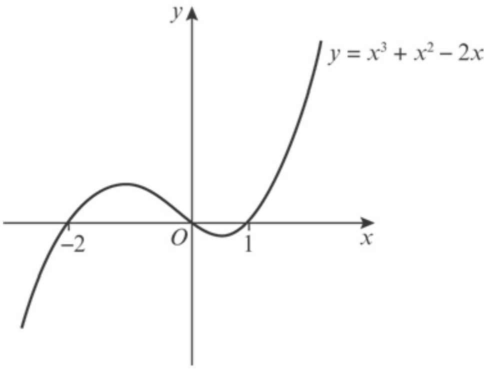
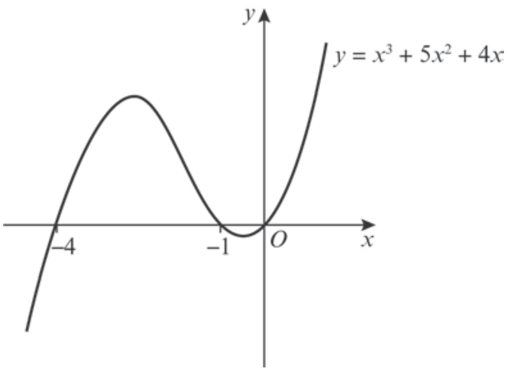
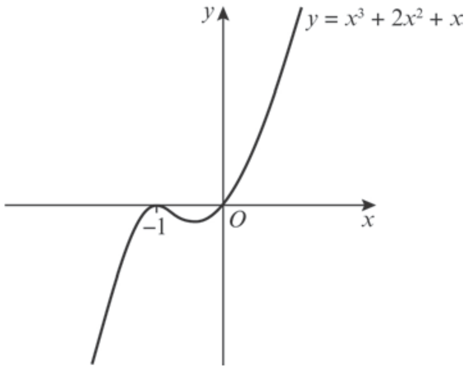
3 **a** $y = x^3 + x^2 - 2x$
$= x(x^2 + x - 2)$
$= x(x + 2)(x - 1)$
$0 = x(x + 2)(x - 1)$
So $x = 0$, $x = -2$ or $x = 1$
So the curve crosses the x-axis at (0, 0), (−2, 0) and (1, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



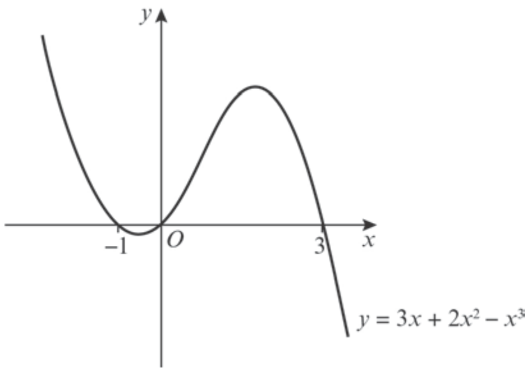
b $y = x^3 + 5x^2 + 4x$
$= x(x^2 + 5x + 4)$
$= x(x + 4)(x + 1)$
$0 = x(x + 4)(x + 1)$
So $x = 0$, $x = -4$ or $x = -1$
So the curve crosses the x-axis at (0, 0), (−4, 0) and (−1, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



c $y = x^3 + 2x^2 + x$
$= x(x^2 + 2x + 1)$
$= x(x + 1)^2$
$0 = x(x + 1)^2$
So $x = 0$ or $x = -1$
So the curve crosses the x-axis at (0, 0) and touches the x-axis at (−1, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



d $y = 3x + 2x^2 - x^3$
$= x(3 + 2x - x^2)$
$= x(3 - x)(1 + x)$
$0 = x(3 - x)(1 + x)$
So $x = 0$, $x = 3$ or $x = -1$
So the curve crosses the x-axis at (0, 0), (3, 0) and (−1, 0).
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



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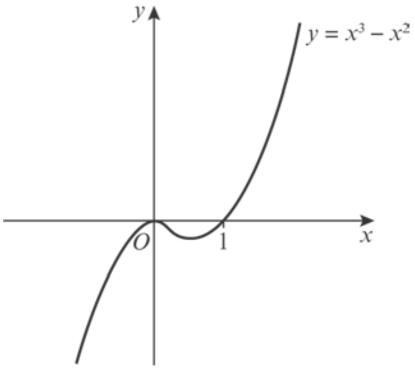
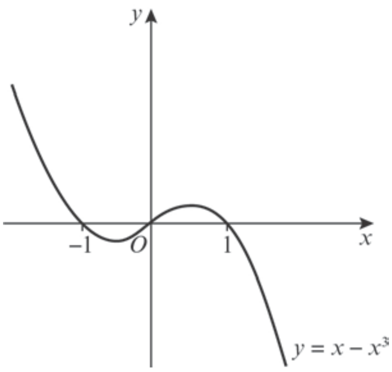
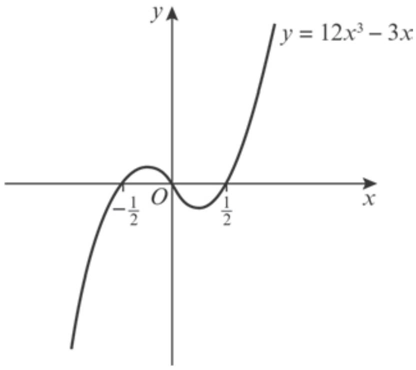
3 e $y = x^3 - x^2$
$= x^2(x - 1)$
$0 = x^2(x - 1)$
So $x = 0$ or $x = 1$
So the curve crosses the x-axis at (1, 0) and touches the x-axis at 0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



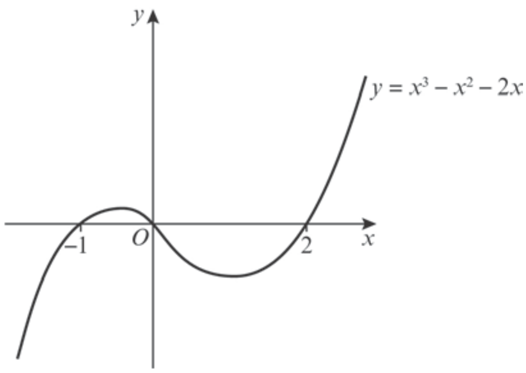
f $y = x - x^3$
$= x(1 - x^2)$
$= x(1 - x)(1 + x)$
$0 = x(1 - x)(1 + x)$
So $x = 0$, $x = 1$ or $x = -1$
So the curve crosses the x-axis at (0, 0), (1, 0) and (−1, 0).
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



g $y = 12x^3 - 3x$
$= 3x(4x^2 - 1)$
$= 3x(2x - 1)(2x + 1)$
$0 = 3x(2x - 1)(2x + 1)$
So $x = 0$, $x = \frac{1}{2}$ or $x = -\frac{1}{2}$
So the curve crosses the x-axis at (0, 0), $\left(\frac{1}{2}, 0\right)$ and $\left(-\frac{1}{2}, 0\right)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



h $y = x^3 - x^2 - 2x$
$= x(x^2 - x - 2)$
$= x(x + 1)(x - 2)$
$0 = x(x + 1)(x - 2)$
So $x = 0$, $x = -1$ or $x = 2$
So the curve crosses the x-axis at (0, 0), (−1, 0) and (2, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



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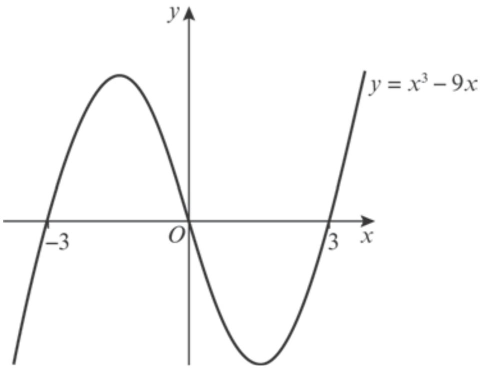
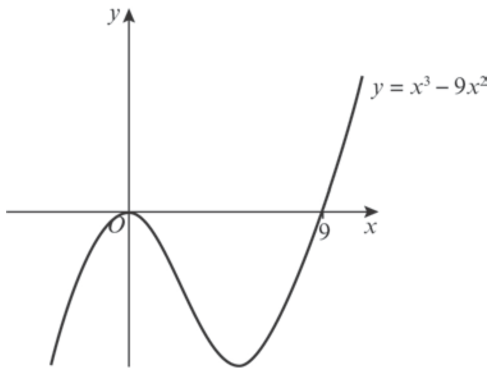
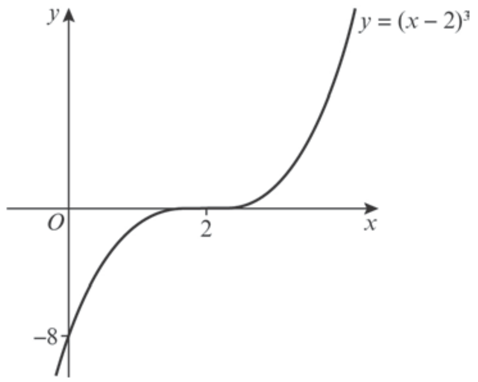
3 i $y = x^3 - 9x$
$= x(x^2 - 9)$
$= x(x - 3)(x + 3)$
$0 = x(x - 3)(x + 3)$
So $x = 0$, $x = 3$ or $x = -3$
So the curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



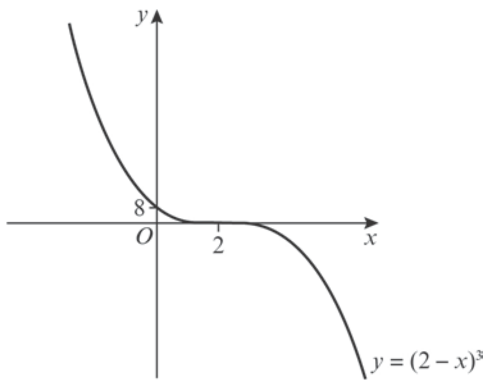
j $y = x^3 - 9x^2$
$= x^2(x - 9)$
$0 = x^2(x - 9)$
So $x = 0$ or $x = 9$
So the curve crosses the x-axis at $(0, 0)$ and touches the x-axis at 0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



4 a $y = (x - 2)^3$
$0 = (x - 2)^3$
So $x = 2$ and the curve crosses the x-axis at $(2, 0)$ only.
When $x = 0$, $y = (-2)^3 = -8$
So the curve crosses the y-axis at $(0, -8)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $y = (2 - x)^3$
$0 = (2 - x)^3$
So $x = 2$ and the curve crosses the x-axis at $(2, 0)$ only.
When $x = 0$, $y = 2^3 = 8$
So the curve crosses y-axis at $(0, 8)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



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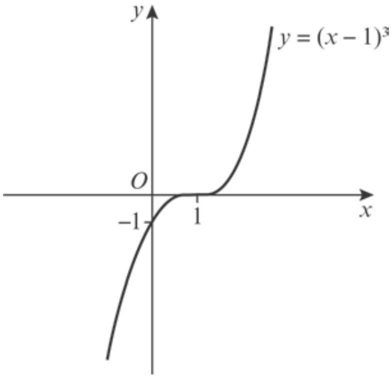
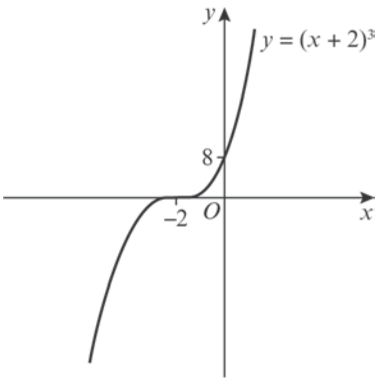
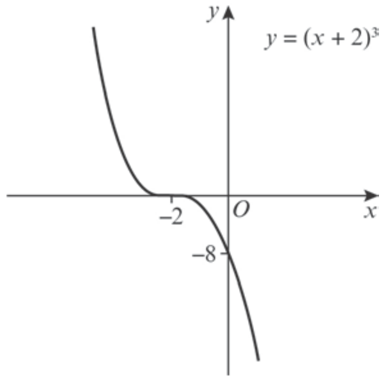
4 c $y = (x-1)^3$
$0 = (x-1)^3$
So $x = 1$ and the curve crosses the x-axis at $(1, 0)$ only.
When $x = 0$, $y = (-1)^3 = -1$
So the curve crosses y-axis at $(0, -1)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



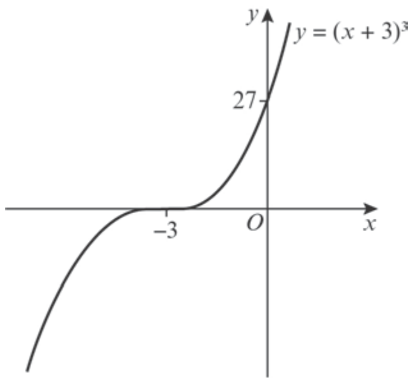
d $y = (x+2)^3$
$0 = (x+2)^3$
So $x = -2$ and the curve crosses the x-axis at $(-2, 0)$ only.
When $x = 0$, $y = 2^3 = 8$
So the curve crosses y-axis at $(0, 8)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



e $y = -(x+2)^3$
$0 = -(x+2)^3$
So $x = -2$ and the curve crosses the x-axis at $(-2, 0)$ only.
When $x = 0$, $y = -2^3 = 8$
So the curve crosses the y-axis at $(0, -8)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



f $y = (x+3)^3$
$0 = (x+3)^3$
So $x = -3$ and the curve crosses the x-axis at $(-3, 0)$ only.
When $x = 0$, $y = 3^3 = 27$
So the curve crosses the y-axis at $(0, 27)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



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4 g $y = (x-3)^3$

$0 = (x-3)^3$

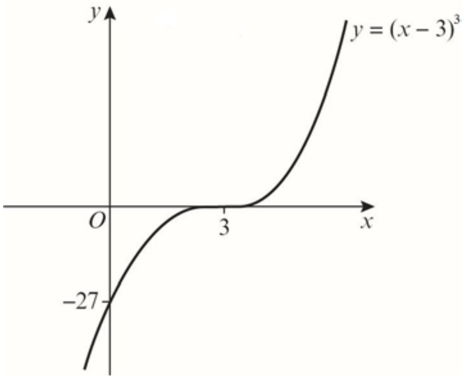
So $x = 3$ and the curve crosses the x-axis at $(3, 0)$ only.

When $x = 0$, $y = (-3)^3 = -27$

So the curve crosses the y-axis at $(0, -27)$.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



h $y = (1-x)^3$

$0 = (1-x)^3$

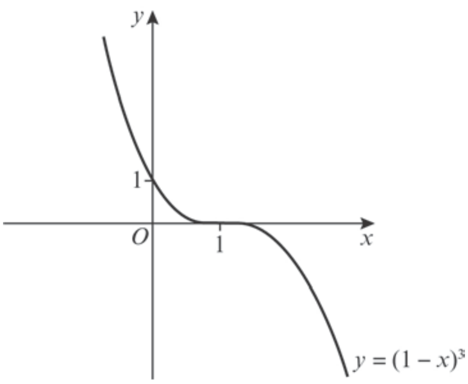
So $x = 1$ and the curve crosses the x-axis at $(1, 0)$ only.

When $x = 0$, $y = 1^3 = 1$

So the curve crosses the y-axis at $(0, 1)$.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



i $y = -(x-2)^3$

$0 = -(x-2)^3$

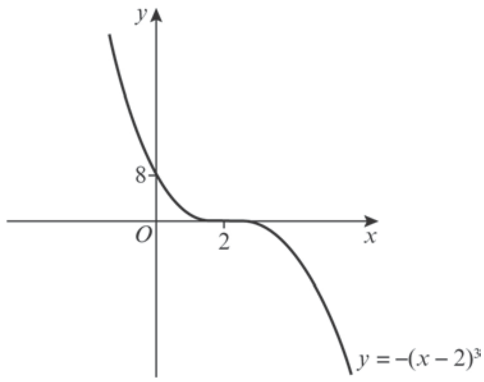
So $x = 2$ and the curve crosses the x-axis at $(2, 0)$ only.

When $x = 0$, $y = -(-2)^3 = 8$

So the curve crosses the y-axis at $(0, 8)$.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



j $y = -\left(x - \frac{1}{2}\right)^3$

$0 = -\left(x - \frac{1}{2}\right)^3$

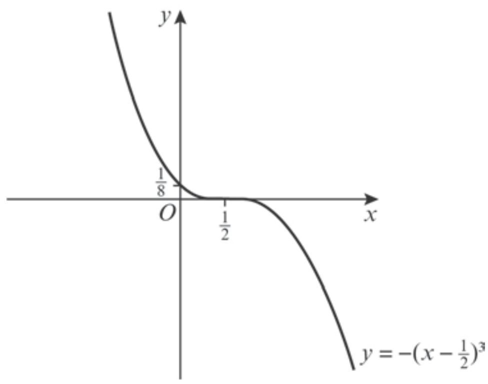
So $x = \frac{1}{2}$ and the curve crosses the x-axis at $(\frac{1}{2}, 0)$ only.

When $x = 0$, $y = -\left(-\frac{1}{2}\right)^3 = \frac{1}{8}$

So the curve crosses the y-axis at $\left(0, \frac{1}{8}\right)$.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



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5 a $y = x^3 + bx^2 + cx + d$

$y = (x + 3)(x + 2)(x - 1)$

$= (x + 3)(x^2 + x - 2)$

$= x^3 + 4x^2 + x - 6$

$b = 4$, $c = 1$, $d = -6$

b When $x = 0$, $y = -6$

So the curve crosses the y-axis at $(0, -6)$.

6 $y = ax^3 + bx^2 + cx + d$

$y = a(x + 1)(x - 2)(x - 3)$

$= a(x + 1)(x^2 - 5x + 6)$

$= a(x^3 - 4x^2 + x + 6)$

The curve crosses the y-axis at $(0, 2)$, so when $x = 0$, $y = 2$.

$2 = a(0 - 0 + 0 + 6)$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x^3 - 4x^2 + x + 6)$

$= \frac{1}{3}x^3 - \frac{4}{3}x^2 + \frac{1}{3}x + 2$

$a = \frac{1}{3}$, $b = -\frac{4}{3}$, $c = \frac{1}{3}$, $d = 2$

7 a $f(x) = (x - 10)(x^2 - 2x) + 12x$

$= x^3 - 12x^2 + 20x + 12x$

$= x^3 - 12x^2 + 32x$

$= x(x^2 - 12x + 32)$

b $f(x) = x(x^2 - 12x + 32)$

$= x(x - 4)(x - 8)$

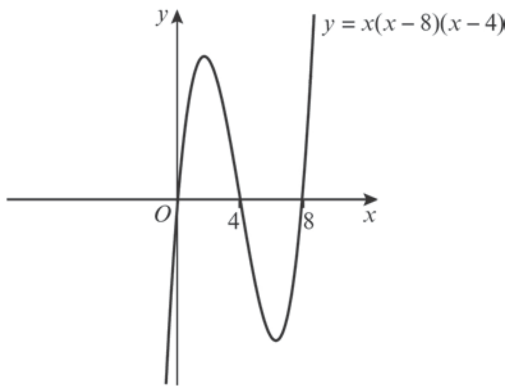
c $0 = x(x - 4)(x - 8)$

So $x = 0$, $x = 4$ or $x = 8$

So the curve crosses the x-axis at $(0, 0)$, $(4, 0)$ and $(8, 0)$

$x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to -\infty$



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