Quadratics 2H

1 a The bridge is 200 m above ground level, since this is the height at the centre of the bridge.

b $0.000\,12x^2 + 200 = 346$

$0.000\,12x^2 = 146$

$x^2 = \dfrac{146}{0.000\,12}$

$x = \pm\sqrt{\dfrac{146}{0.000\,12}}$

So $x = 1103$ and $x = -1103$

c length = 1103 × 2 = 2206 m

2 a $-0.01x^2 + 0.975x + 16 = 32.5$

$-0.01x^2 + 0.975x - 16.5 = 0$

Using the formula, where $a = -0.01$, $b = 0.975$ and $c = -16.5$,

$$x = \frac{-0.975 \pm \sqrt{0.975^2 - 4(-0.01)(-16.5)}}{2(-0.01)}$$

$$x = \frac{0.975 \pm \sqrt{0.290\,625}}{0.02}$$

$x = 75.7$ and $x = 21.8$ (to 3 s.f.)

21.8 mph and 75.7 mph

b $y = -0.01x^2 + 0.975x + 16$

$= -0.01(x^2 - 97.5x) + 16$

$= -0.01((x - 48.75)^2 - 2376.5625) + 16$

$= -0.01(x - 48.75)^2 + 39.765\,625$

$A = 39.77$ (to 4 s.f.), $B = 0.01$ and $C = 48.75$

c The greatest fuel efficiency is the maximum, when $x = 48.75$

48.75 mph

d When $x = 120$,

$y = -0.01(120)^2 + 0.975(120) + 16$

$= -11$

A negative fuel consumption is impossible, so this model is not valid for very high speeds.

3 a Without any fertliser, $f = 0$, so each hectare would yield 6 tonnes of grain.

3 b When $f = 20$,

$g = 6 + 0.03(20) - 0.00006(20)^2$

$= 6.576$

For an extra tonne yield, $g = 6.576 + 1$

$= 7.576$

$6 + 0.03f - 0.000\,06f^2 = 7.576$

$1.576 - 0.03f + 0.000\,06f^2 = 0$

Using the formula, where $a = 0.000\,06$, $b = -0.03$ and $c = 1.576$,

$$x = \frac{-(-0.03) \pm \sqrt{(-0.03)^2 - 4(0.000\,06)(1.576)}}{2(0.000\,06)}$$

$$x = \frac{0.03 \pm \sqrt{0.000\,521\,76}}{0.000\,12}$$

$x = 440.4$ and $x = 59.6$ (to 1 d.p.)

$59.6 - 20 = 39.6$

39.6 kilograms per hectare

4 a $t = M - 1000p$, $t = 10\,000$ when $p = £30$

$10\,000 = M - 1000 \times 30$

$M = 40\,000$

b $r = p(40\,000 - 1000p)$

$= -1000p^2 + 40\,000p$

$= -1000(p^2 - 40p)$

$= -1000((p - 20)^2 - 400)$

$= -1000(p - 20)^2 + 400\,000$

$A = 400\,000$, $B = 1000$ and $C = 20$

c $r = = -1000(p - 20)^2 + 400\,000$

maximum = £400 000 when $p = 20$

They should charge £20 per ticket.

Challenge

a $d(s) = as^2 + bs + c$
When $s = 20$, $d = 6$:
$6 = a(20)^2 + b(20) + c$
$6 = 400a + 20b + c$ (**1**)

When $s = 30$, d = 14:
$14 = a(30)^2 + b(30) + c$
$14 = 900a + 30b + c$ (**2**)

When $s = 20$, $d = 24$:
$24 = a(40)^2 + b(40) + c$
$24 = 1600a + 40b + c$ (**3**)

(**2**) − (**1**):
$(14 = 900a + 30b + c) - (6 = 400a + 20b + c)$
$\Rightarrow 8 = 500a + 10b$ (**4**)

(**3**) − (**1**):
$(24 = 1600a + 40b + c) - (6 = 400a + 20b + c)$
$\Rightarrow 18 = 1200a + 20b$ (**5**)

(**5**) − 2 × (**4**):
$(18 = 1200a + 20b) - 2(8 = 500a + 10b)$
$\Rightarrow 2 = 200a$, so $a = 0.01$

$8 = 500(0.01) + 10b$
$8 = 5 + 10b \Rightarrow b = 0.3$
$6 = 400(0.01) + 20(0.3) + c$
$6 = 4 + 6 + c \Rightarrow c = -4$
$a = 0.01$, $b = 0.3$ and $c = -4$

b $0.01s^2 + 0.3s - 4 = 20$
$0.01s^2 + 0.3s - 24 = 0$
Using the formula, where $a = 0.01$, $b = 0.3$ and $c = -24$,

$$x = \frac{-0.3 \pm \sqrt{0.3^2 - 4(0.01)(-24)}}{2(0.01)}$$

$$x = \frac{-0.3 \pm \sqrt{1.05}}{0.02}$$

$x = 36.2$ or -66.2 (to 3 s.f.)
The speed of the car must be positive, so is 36.2 mph.