Quadratics 2F

1 a $y = x^2 - 6x + 8$

As $a = 1$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = 8$, so the graph crosses the y-axis at (0, 8).

When $y = 0$,

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

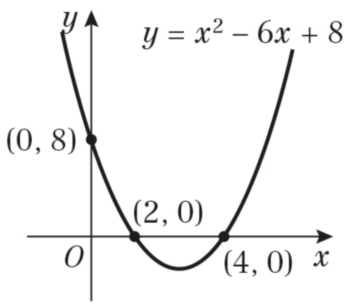
$x = 2$ or $x = 4$, so the graph crosses the x-axis at (2, 0) and (4, 0).

Completing the square:

$$x^2 - 6x + 8 = (x - 3)^2 - 9 + 8$$
$$= (x - 3)^2 - 1$$

So the minimum point has coordinate (3, −1).

The sketch of the graph is:



b $y = x^2 + 2x - 15$

As $a = 1$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = -15$, so the graph crosses the y-axis at (0, −15).

When $y = 0$,

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

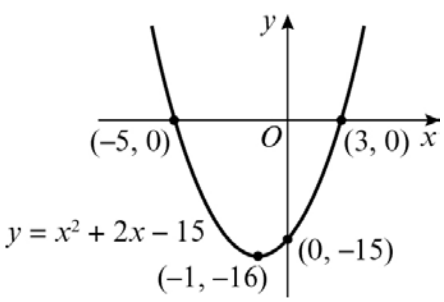
$x = 3$ or $x = -5$, so the graph crosses the x-axis at (3, 0) and (−5, 0).

Completing the square:

$$x^2 + 2x - 15 = (x + 1)^2 - 1 - 15$$
$$= (x + 1)^2 - 16$$

So the minimum point has coordinate (−1, −16).

The sketch of the graph is:



c $y = 25 - x^2$

As $a = -1$ is negative, the graph has a ∩ shape and a maximum point.

When $x = 0$, $y = 25$, so the graph crosses the y-axis at (0, 25).

When $y = 0$,

$25 - x^2 = 0$

$(5 + x)(5 - x) = 0$

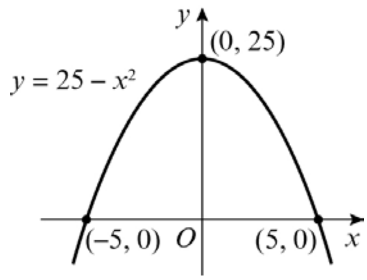
$x = -5$ or $x = 5$, so the graph crosses the x-axis at (−5, 0) and (5, 0).

Completing the square:

$$25 - x^2 = -x^2 + 0x + 25$$
$$= -(x^2 - 0x - 25)$$
$$= -(x - 0)^2 + 25$$

So the maximum point has coordinate (0, 25).

The sketch of the graph is:



d $y = x^2 + 3x + 2$

As $a = 1$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = 2$, so the graph crosses the y-axis at (0, 2).

When $y = 0$,

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

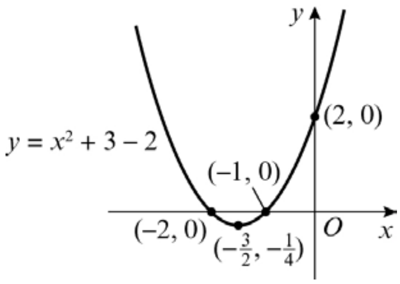
$x = -2$ or $x = -1$, so the graph crosses the x-axis at (−2, 0) and (−1, 0).

Completing the square:

$$x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2$$
$$= \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$$

So the minimum point has coordinate $\left(-\frac{3}{2}, -\frac{1}{4}\right)$.

1 d The sketch of the graph is:



e $y = -x^2 + 6x + 7$

As $a = -1$ is negative, the graph has a ∩ shape and a maximum point.

When $x = 0$, $y = 7$, so the graph crosses the y-axis at (0, 7).

When $y = 0$,

$-x^2 + 6x + 7 = 0$

$(-x - 1)(x - 7) = 0$

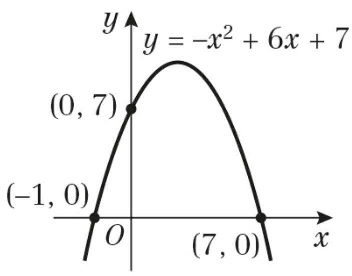
$x = -1$ or $x = 7$, so the graph crosses the x-axis at (−1, 0) and (7, 0).

Completing the square:

$$-x^2 + 6x + 7 = -(x^2 - 6x) + 7$$
$$= -((x-3)^2 - 9) + 7$$
$$= -(x-3)^2 + 16$$

So the maximum point has coordinate (3, 16).

The sketch of the graph is:



f $y = 2x^2 + 4x + 10$

As $a = 2$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = 10$, so the graph crosses the y-axis at (0, 10).

When $y = 0$,

$2x^2 + 4x + 10 = 0$

Using the quadratic formula,

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(10)}}{2 \times 2}$$

$$x = \frac{-4 \pm \sqrt{-64}}{4}$$

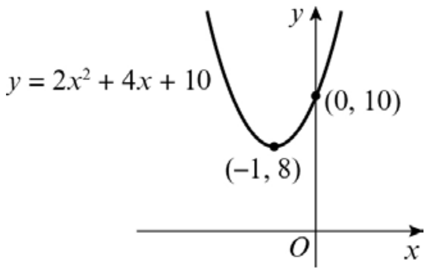
There are no real solutions, so the graph does not cross the x-axis.

f Completing the square:

$$2x^2 + 4x + 10 = 2(x^2 + 2x) + 10$$
$$= 2((x + 1)^2 - 1) + 10$$
$$= 2(x + 1)^2 + 8$$

So the minimum point has coordinate (−1, 8).

The sketch of the graph is:



g $y = 2x^2 + 7x - 15$

As $a = 2$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = -15$, so the graph crosses the y-axis at (0, −15).

When $y = 0$,

$2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

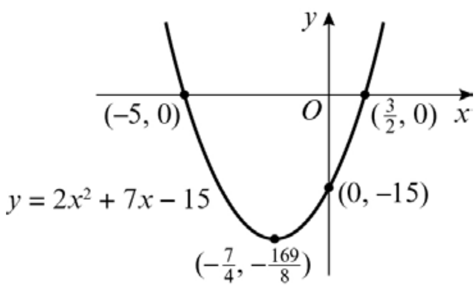
$x = \frac{3}{2}$ or $x = -5$, so the graph crosses the x-axis at $\left(\frac{3}{2}, 0\right)$ and (−5, 0).

Completing the square:

$$2x^2 + 7x - 15 = 2\left(x^2 + \frac{7}{2}x\right) - 15$$
$$= 2\left(\left(x + \frac{7}{4}\right)^2 - \frac{49}{16}\right) - 15$$
$$= 2\left(x + \frac{7}{4}\right)^2 - \frac{169}{8}$$

So the minimum point has coordinate $\left(-\frac{7}{4}, -\frac{169}{8}\right)$.

The sketch of the graph is:



1 h $y = 6x^2 - 19x + 10$

As $a = 6$ is positive, the graph has a $\cup$ shape and a minimum point.

When $x = 0$, $y = 10$, so the graph crosses the y-axis at (0, 10).

When $y = 0$,

$6x^2 - 19x + 10 = 0$

$(3x - 2)(2x - 5) = 0$

$x = \frac{2}{3}$ or $x = \frac{5}{2}$, so the graph crosses the x-axis at $\left(\frac{2}{3}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$.

Completing the square:

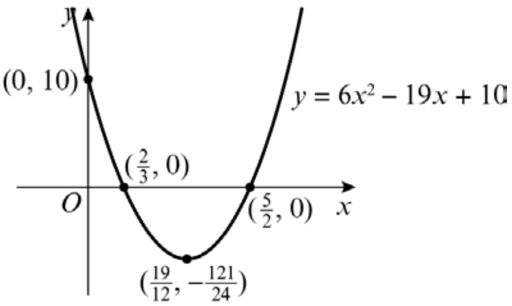
$$6x^2 - 19x + 10 = 6\left(x^2 - \frac{19}{6}x\right) + 10$$

$$= 6\left(\left(x - \frac{19}{12}\right)^2 - \frac{361}{144}\right) + 10$$

$$= 6\left(x - \frac{19}{12}\right)^2 - \frac{121}{24}$$

So the minimum point has coordinate $\left(\frac{19}{12}, -\frac{121}{24}\right)$.

The sketch of the graph is:



i $y = 4 - 7x - 2x^2$

As $a = -2$ is negative, the graph has a $\cap$ shape and a maximum point.

When $x = 0$, $y = 4$, so the graph crosses the y-axis at (0, 4).

When $y = 0$,

$-2x^2 - 7x + 4 = 0$

$(-2x + 1)(x + 4) = 0$

$x = \frac{1}{2}$ or $x = -4$, so the graph crosses the x-axis at $\left(\frac{1}{2}, 0\right)$ and (−4, 0).

Completing the square:

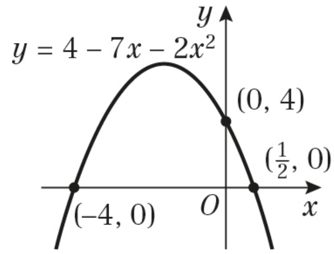
$$-2x^2 - 7x + 4 = -2\left(x^2 + \frac{7}{2}x\right) + 4$$

$$= -2\left(\left(x + \frac{7}{4}x\right)^2 - \frac{49}{16}\right) + 4$$

$$= -2\left(x + \frac{7}{4}x\right)^2 + \frac{81}{8}$$

i So the maximum point has coordinate $\left(-\frac{7}{4}, \frac{81}{8}\right)$.

The sketch of the graph is:



j $y = 0.5x^2 + 0.2x + 0.02$

As a = 0.5 is positive, the graph has a $\cup$ shape and a minimum point.

When $x = 0$, $y = 0.02$, so the graph crosses the y-axis at (0, 0.02).

When $y = 0$,

$0.5x^2 + 0.2x + 0.02 = 0$

Using the quadratic formula,

$$x = \frac{-0.2 \pm \sqrt{0.2^2 - 4(0.5)(0.02)}}{2 \times 0.5}$$

$$x = -0.2 \pm \sqrt{0}$$

$$= -0.2$$

There is only one solution, so the graph touches the x-axis.

Completing the square:

$0.5x^2 + 0.2x + 0.02$

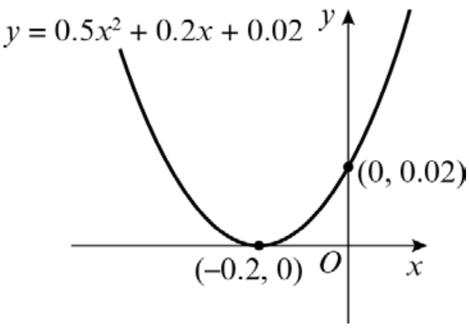
$= 0.5(x^2 + 0.4x) + 0.02$

$= 0.5((x + 0.2)^2 - 0.04) + 0.02$

$= 0.5(x + 0.2)^2$

So the minimum point has coordinate (−0.2, 0).

The sketch of the graph is:



2 a The graph crosses the y-axis at (0, 15), so $c = 15$.
The graph crosses the x-axis at (3, 0) and (5, 0) and has a minimum value.
$(x - 3)(x - 5) = 0$
$x^2 - 8x + 15 = 0$
$a = 1$, $b = -8$ and $c = 15$

b The graph crosses the y-axis at (0, 10), so $c = 10$.
The graph crosses the x-axis at (−2, 0) and (5, 0) and has a maximum value.
$-(x + 2)(x - 5) = 0$
$-x^2 + 3x + 10 = 0$
$a = -1$, $b = 3$ and $c = 10$

c The graph crosses the y-axis at (0, −18), so $c = -18$.
The graph crosses the x-axis at (−3, 0) and (3, 0) and has a minimum value.
$(x + 3)(x - 3) = 0$
$x^2 + 0x - 9 = 0$
But $c = -18$, not -9, so $2(x^2 + 0x - 9) = 0$
$a = 2$, $b = 0$ and $c = -18$

d The graph crosses the y-axis at (0, −1), so $c = -1$.
The graph crosses the x-axis at (−1, 0) and (4, 0) and has a minimum value.
$(x + 1)(x - 4) = 0$
$x^2 - 3x - 4 = 0$
But $c = -1$, not -4, so $\frac{1}{4}(x^2 - 3x - 4) = 0$
$a = \frac{1}{4}$, $b = -\frac{3}{4}$ and $c = -1$

3 Minimum value = (5, −3), so the line of symmetry is at $x = 5$.
The reflection of (4, 0) in the line $y = 5$ is (6, 0).
$(x - 6)(x - 4) = 0$
$x^2 - 10x + 24 = 0$
Completing the square:
$x^2 - 10x + 24 = (x - 5)^2 - 25 + 24$
$= (x - 5)^2 - 1$
But the minimum value is (5, − 3), therefore:
$y = 3(x - 5)^2 - 3$
$= 3x^2 - 30x + 72$
$a = 3$, $b = -30$ and $c = 72$