

## Algebraic expressions, Mixed Exercise 1

$$\begin{aligned} 1 \text{ a } & y^3 \times y^5 \\ & = y^{3+5} \\ & = y^8 \end{aligned}$$

$$\begin{aligned} \text{b } & 3x^2 \times 2x^5 \\ & = 3 \times 2 \times x^{2+5} \\ & = 6x^7 \end{aligned}$$

$$\begin{aligned} \text{c } & (4x^2)^3 \div 2x^5 \\ & = 4^3 x^{2 \times 3} \div 2x^5 \\ & = 64x^6 \div 2x^5 \\ & = 32x^{6-5} \\ & = 32x \end{aligned}$$

$$\begin{aligned} \text{d } & 4b^2 \times 3b^3 \times b^4 \\ & = 4 \times 3 \times b^{2+3+4} \\ & = 12b^9 \end{aligned}$$

$$\begin{aligned} 2 \text{ a } & (x+3)(x-5) \\ & = x^2 - 5x + 3x - 15 \\ & = x^2 - 2x - 15 \end{aligned}$$

$$\begin{aligned} \text{b } & (2x-7)(3x+1) \\ & = 6x^2 + 2x - 21x - 7 \\ & = 6x^2 - 19x - 7 \end{aligned}$$

$$\begin{aligned} \text{c } & (2x+5)(3x-y+2) \\ & = 2x(3x-y+2) + 5(3x-y+2) \\ & = 6x^2 - 2xy + 4x + 15x - 5y + 10 \\ & = 6x^2 - 2xy + 19x - 5y + 10 \end{aligned}$$

$$\begin{aligned} 3 \text{ a } & x(x+4)(x-1) \\ & = (x^2+4x)(x-1) \\ & = x^3 - x^2 + 4x^2 - 4x \\ & = x^3 + 3x^2 - 4x \end{aligned}$$

$$\begin{aligned} \text{b } & (x+2)(x-3)(x+7) \\ & = (x^2-3x+2x-6)(x+7) \\ & = (x^2-x-6)(x+7) \\ & = x^2(x+7) - x(x+7) - 6(x+7) \\ & = x^3 + 7x^2 - x^2 - 7x - 6x - 42 \\ & = x^3 + 6x^2 - 13x - 42 \end{aligned}$$

$$\begin{aligned} \text{c } & (2x+3)(x-2)(3x-1) \\ & = (2x^2-4x+3x-6)(3x-1) \\ & = (2x^2-x-6)(3x-1) \\ & = 2x^2(3x-1) - x(3x-1) - 6(3x-1) \\ & = 6x^3 - 2x^2 - 3x^2 + x - 18x + 6 \\ & = 6x^3 - 5x^2 - 17x + 6 \end{aligned}$$

$$\begin{aligned} 4 \text{ a } & 3(5y+4) \\ & = 15y + 12 \end{aligned}$$

$$\begin{aligned} \text{b } & 5x^2(3-5x+2x^2) \\ & = 15x^2 - 25x^3 + 10x^4 \end{aligned}$$

$$\begin{aligned} \text{c } & 5x(2x+3) - 2x(1-3x) \\ & = 10x^2 + 15x - 2x + 6x^2 \\ & = 16x^2 + 13x \end{aligned}$$

$$\begin{aligned} \text{d } & 3x^2(1+3x) - 2x(3x-2) \\ & = 3x^2 + 9x^3 - 6x^2 + 4x \\ & = 9x^3 - 3x^2 + 4x \end{aligned}$$

$$\begin{aligned} 5 \text{ a } & 3x^2 + 4x \\ & = x(3x+4) \end{aligned}$$

$$\begin{aligned} \text{b } & 4y^2 + 10y \\ & = 2y(2y+5) \end{aligned}$$

$$\begin{aligned} \text{c } & x^2 + xy + xy^2 \\ & = x(x+y+y^2) \end{aligned}$$

$$\begin{aligned} \text{d } & 8xy^2 + 10x^2y \\ & = 2xy(4y+5x) \end{aligned}$$

$$\begin{aligned} 6 \text{ a } & x^2 + 3x + 2 \\ & = x^2 + x + 2x + 2 \\ & = x(x+1) + 2(x+1) \\ & = (x+1)(x+2) \end{aligned}$$

$$\begin{aligned} \text{b } & 3x^2 + 6x \\ & = 3x(x+2) \end{aligned}$$

$$\begin{aligned} \text{c } & x^2 - 2x - 35 \\ & = x^2 - 7x + 5x - 35 \\ & = x(x-7) + 5(x-7) \\ & = (x-7)(x+5) \end{aligned}$$

$$\begin{aligned} \text{d } & 2x^2 - x - 3 \\ & = 2x^2 - 3x + 2x - 3 \\ & = x(2x-3) + (2x-3) \\ & = (2x-3)(x+1) \end{aligned}$$

$$\begin{aligned} \text{e } & 5x^2 - 13x - 6 \\ & = 5x^2 + 2x - 15x - 6 \\ & = x(5x+2) - 3(5x+2) \\ & = (5x+2)(x-3) \end{aligned}$$

$$\begin{aligned}
 \mathbf{6 f} \quad & 6 - 5x - x^2 \\
 & = 6 + x - 6x - x^2 \\
 & = (6 + x) - x(6 + x) \\
 & = (6 + x)(1 - x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7 a} \quad & 2x^3 + 6x \\
 & = 2x(x^2 + 3)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & x^3 - 36x \\
 & = x(x^2 - 36) \\
 & = x(x^2 - 6^2) \\
 & = x(x + 6)(x - 6)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & 2x^3 + 7x^2 - 15x \\
 & = x(2x^2 + 7x - 15) \\
 & = x(2x^2 - 3x + 10x - 15) \\
 & = x(x(2x - 3) + 5(2x - 3)) \\
 & = x(2x - 3)(x + 5)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{8 a} \quad & 9x^3 \div 3x^{-3} \\
 & = 3x^{3 - (-3)} \\
 & = 3x^6
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \left(4^{\frac{3}{2}}\right)^{\frac{1}{3}} \\
 & = 4^{\frac{3}{2} \times \frac{1}{3}} \\
 & = 4^{\frac{1}{2}} \\
 & = \sqrt{4} \\
 & = \pm 2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & 3x^{-2} \times 2x^4 \\
 & = 6x^{-2+4} \\
 & = 6x^2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad & 3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}} \\
 & = \frac{1}{2}x^{\frac{1}{3} - \frac{2}{3}} \\
 & = \frac{1}{2}x^{-\frac{1}{3}} \text{ or } \frac{1}{2(\sqrt[3]{x})}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9 a} \quad & \left(\frac{8}{27}\right)^{\frac{2}{3}} \\
 & = \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9 a} \quad & \left(\frac{8}{27}\right)^{\frac{2}{3}} \\
 & = \left(\frac{2}{3}\right)^2 \\
 & = \frac{4}{9}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \left(\frac{225}{289}\right)^{\frac{3}{2}} \\
 & = \left(\frac{\sqrt{225}}{\sqrt{289}}\right)^3 \\
 & = \pm \frac{15^3}{17^3} \\
 & = \pm \frac{3375}{4913}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10 a} \quad & \frac{3}{\sqrt{63}} \\
 & = \frac{3}{\sqrt{9 \times 7}} \\
 & = \frac{3}{3\sqrt{7}} \\
 & = \frac{1}{\sqrt{7}} \\
 & = \frac{\sqrt{7}}{7}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \sqrt{20} + 2\sqrt{45} - \sqrt{80} \\
 & = \sqrt{4}\sqrt{5} + 2\sqrt{9}\sqrt{5} - \sqrt{16}\sqrt{5} \\
 & = 2\sqrt{5} + 2 \times 3\sqrt{5} - 4\sqrt{5} \\
 & = \sqrt{5}(2 + 6 - 4) \\
 & = 4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11 a} \quad & \text{When } x = 25, \\
 & 35x^2 + 2x - 48 \\
 & = 35 \times 25^2 + 2 \times 25 - 48 \\
 & = 21\,877
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & 35x^2 + 2x - 48 \\
 & = 35x^2 + 42x - 40x - 48 \\
 & = 7x(5x + 6) - 8(5x + 6) \\
 & = (5x + 6)(7x - 8) \\
 & \text{When } x = 25, 5x + 6 = 131 \\
 & \text{and } 7x - 8 = 167; \\
 & \text{both 131 and 167 are prime numbers.}
 \end{aligned}$$

$$\begin{aligned} \mathbf{12\ a} \quad & \sqrt{2}(3+\sqrt{5}) \\ & = 3\sqrt{2} + \sqrt{2}\sqrt{5} \\ & = 3\sqrt{2} + \sqrt{10} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & (2-\sqrt{5})(5+\sqrt{3}) \\ & = 10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{3}\sqrt{5} \\ & = 10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & (6-\sqrt{2})(4-\sqrt{7}) \\ & = 24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{2}\sqrt{7} \\ & = 24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14} \end{aligned}$$

$$\begin{aligned} \mathbf{13\ a} \quad & \frac{1}{\sqrt{3}} \\ & = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ & = \frac{\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \frac{1}{\sqrt{2}-1} \\ & = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} \\ & = \frac{\sqrt{2}+1}{2-1} \\ & = \sqrt{2}+1 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & \frac{3}{\sqrt{3}-2} \\ & = \frac{3(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} \\ & = \frac{3\sqrt{3}+6}{3-4} \\ & = \frac{3\sqrt{3}+6}{-1} \\ & = -3\sqrt{3}-6 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad & \frac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}} \\ & = \frac{(\sqrt{23}-\sqrt{37})(\sqrt{23}-\sqrt{37})}{(\sqrt{23}+\sqrt{37})(\sqrt{23}-\sqrt{37})} \\ & = \frac{23-2\sqrt{23}\sqrt{37}+37}{23-37} \\ & = \frac{60-2\sqrt{851}}{-14} \\ & = \frac{30-\sqrt{851}}{-7} \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad & \frac{1}{(2+\sqrt{3})^2} \\ & = \frac{1}{(2+\sqrt{3})(2+\sqrt{3})} \\ & = \frac{1}{4+2\sqrt{3}+2\sqrt{3}+\sqrt{9}} \\ & = \frac{1}{4+2\sqrt{3}+2\sqrt{3}+\sqrt{9}} \\ & = \frac{1 \times (7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} \\ & = \frac{7-4\sqrt{3}}{49-28\sqrt{3}+28\sqrt{3}-48} \\ & = \frac{7-4\sqrt{3}}{1} \\ & = 7-4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad & \frac{1}{(4-\sqrt{7})^2} \\ & = \frac{1}{(4-\sqrt{7})(4-\sqrt{7})} \\ & = \frac{1}{16-4\sqrt{7}-4\sqrt{7}+\sqrt{49}} \\ & = \frac{1}{23-8\sqrt{7}} \end{aligned}$$

$$\begin{aligned}
 \mathbf{13 f} \quad & \frac{1}{(4-\sqrt{7})^2} \\
 &= \frac{1 \times (23+8\sqrt{7})}{(23-8\sqrt{7})(23+8\sqrt{7})} \\
 &= \frac{23+8\sqrt{7}}{529+184\sqrt{7}-184\sqrt{7}-448} \\
 &= \frac{23+8\sqrt{7}}{81}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{14 a} \quad & x^3 - x^2 - 17x - 15 \\
 &= (x+3)(x^2 + bx + c) \\
 &= x^3 + bx^2 + cx + 3x^2 + 3bx + 3c \\
 &= x^3 + bx^2 + 3x^2 + 3bx + cx + 3c \\
 &= x^3 - x^2 - 17x - 15 \\
 &\text{Equating the coefficients gives } b + 3 = -1, \\
 &3b + c = -17, 3c = -15 \\
 &\text{Using } b + 3 = -1, \quad b = -4 \\
 &\text{Using } 3c = -15, \quad c = -5
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & x^3 - x^2 - 17x - 15 \\
 &= (x+3)(x^2 - 4x - 5) \\
 &= (x+3)(x^2 - 5x + x - 5) \\
 &= (x+3)(x(x-5) + (x-5)) \\
 &= (x+3)(x-5)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{15 a} \quad & y^{\frac{1}{3}} \\
 &= \left(\frac{1}{64}x^3\right)^{\frac{1}{3}} \\
 &= \frac{1}{\sqrt[3]{64}}x^{3 \times \frac{1}{3}} \\
 &= \frac{x}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & 4y^{-1} \\
 &= 4\left(\frac{1}{64}x^3\right)^{-1} \\
 &= 4 \times \frac{1}{\frac{1}{64}}x^{3 \times (-1)} \\
 &= 256x^{-3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{16} \quad & \frac{5}{\sqrt{75}-\sqrt{50}} \\
 &= \frac{5}{\sqrt{25 \times 3}-\sqrt{25 \times 2}} \\
 &= \frac{5}{5\sqrt{3}-5\sqrt{2}} \\
 &= \frac{1}{\sqrt{3}-\sqrt{2}} \\
 &= \frac{1 \times (\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} \\
 &= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{9}+\sqrt{2}\sqrt{3}-\sqrt{2}\sqrt{3}-\sqrt{4}} \\
 &= \frac{\sqrt{3}+\sqrt{2}}{1} \\
 &= \sqrt{3}+\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{17} \quad & (\sqrt{11}-5)(5-\sqrt{11}) \\
 &= 5\sqrt{11}-\sqrt{121}-25+5\sqrt{11} \\
 &= 10\sqrt{11}-36
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{18} \quad & x - 64x^3 \\
 &= x(1 - 64x^2) \\
 &= x[1^2 - (8x)^2] \\
 &= x(1 + 8x)(1 - 8x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{19} \quad & 27^{2x+1} \\
 &= (3^3)^{2x+1} \\
 &= 3^{3(2x+1)} \\
 &= 3^{6x+3} \\
 &= 3^y \\
 &y = 6x + 3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{20} \quad & 8 + x\sqrt{12} = \frac{8x}{\sqrt{3}} \\
 & 8\sqrt{3} + x\sqrt{12}\sqrt{3} = 8x \\
 & 8\sqrt{3} + x\sqrt{36} = 8x \\
 & 8\sqrt{3} + 6x = 8x \\
 & 8\sqrt{3} = 2x \\
 & x = 4\sqrt{3}
 \end{aligned}$$

**21** Area =  $\sqrt{12}$  cm<sup>2</sup>, length =  $(1 + \sqrt{3})$  cm

$$\begin{aligned} \text{Width} &= \frac{\sqrt{12}}{1 + \sqrt{3}} \\ &= \frac{\sqrt{12} \times (1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} \\ &= \frac{\sqrt{12} - \sqrt{36}}{1 - \sqrt{3} + \sqrt{3} - \sqrt{9}} \\ &= \frac{\sqrt{4 \times 3} - 6}{-2} \\ &= \frac{2\sqrt{3} - 6}{-2} \\ &= \frac{-\sqrt{3} + 3}{1} \\ &= 3 - \sqrt{3} \text{ cm} \end{aligned}$$

**22**

$$\begin{aligned} &\frac{(2 - \sqrt{x})^2}{\sqrt{x}} \\ &= \frac{(2 - x^{\frac{1}{2}})^2}{x^{\frac{1}{2}}} \\ &= \frac{(2 - x^{\frac{1}{2}})(2 - x^{\frac{1}{2}})}{x^{\frac{1}{2}}} \\ &= \frac{4 - 2x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + x^{\frac{1}{2} + \frac{1}{2}}}{x^{\frac{1}{2}}} \\ &= x^{-\frac{1}{2}}(4 - 4x^{\frac{1}{2}} + x) \\ &= 4x^{-\frac{1}{2}} - 4x^{-\frac{1}{2} + \frac{1}{2}} + x^{1 - \frac{1}{2}} \\ &= 4x^{-\frac{1}{2}} - 4x^0 + x^{\frac{1}{2}} \\ &= 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}} \\ &= \frac{4}{\sqrt{x}} - 4 + \sqrt{x} \end{aligned}$$

**23 a**  $243\sqrt{3}$

$$\begin{aligned} &= 3^5 \times 3^{\frac{1}{2}} \\ &= 3^{5 + \frac{1}{2}} \\ &= 3^{\frac{11}{2}} \\ a &= \frac{11}{2} \end{aligned}$$

**b** From part a:  $3^x \times 27^y = 243\sqrt{3} = 3^{\frac{11}{2}}$

$$\begin{aligned} 3^x \times 3^{3y} &= 3^{\frac{11}{2}} \\ 3^{x+3y} &= 3^{\frac{11}{2}} \end{aligned}$$

So  $x + 3y = \frac{11}{2}$

$$\begin{aligned} 3y &= \frac{11 - 2x}{2} \\ y &= \frac{11 - 2x}{6} \end{aligned}$$

**24**

$$\begin{aligned} &\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}} \\ &= \frac{4x^3 + x^{\frac{5}{2}}}{x^{\frac{1}{2}}} \\ &= x^{-\frac{1}{2}}(4x^3 + x^{\frac{5}{2}}) \\ &= 4x^{-\frac{1}{2} + 3} + x^{-\frac{1}{2} + \frac{5}{2}} \\ &= 4x^{\frac{5}{2}} + x^2 \\ &= 4x^a + x^b \\ a &= \frac{5}{2}, b = 2 \end{aligned}$$

**Challenge**

**a**  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$   
 $= a - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} - b$   
 $= a - b$

**b** Rationalising the denominators:

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{24} + \sqrt{25}}$$

$$= \frac{\sqrt{1} - \sqrt{2}}{(\sqrt{1} + \sqrt{2})(\sqrt{1} - \sqrt{2})} + \frac{\sqrt{2} - \sqrt{3}}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} + \frac{\sqrt{3} - \sqrt{4}}{(\sqrt{3} + \sqrt{4})(\sqrt{3} - \sqrt{4})} \dots + \frac{\sqrt{24} - \sqrt{25}}{(\sqrt{24} + \sqrt{25})(\sqrt{24} - \sqrt{25})}$$

$$= \frac{\sqrt{1} - \sqrt{2}}{1 - 2} + \frac{\sqrt{2} - \sqrt{3}}{2 - 3} + \frac{\sqrt{3} - \sqrt{4}}{3 - 4} + \dots + \frac{\sqrt{24} - \sqrt{25}}{24 - 25}$$

$$= -(\sqrt{1} - \sqrt{2}) - (\sqrt{2} - \sqrt{3}) - (\sqrt{3} - \sqrt{4}) - \dots - (\sqrt{24} - \sqrt{25})$$

$$= -\sqrt{1} + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4} - \dots - \sqrt{24} + \sqrt{25}$$

$$= -\sqrt{1} + \sqrt{25}$$

$$= -1 + 5$$

$$= 4$$