

Algebraic expressions 1D

1 a $x^3 \div x^{-2} = x^{3-(-2)}$
 $= x^5$

b $x^5 \div x^7 = x^{5-7}$
 $= x^{-2}$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}} = x^{\frac{3+5}{2}}$
 $= x^4$

d $(x^2)^{\frac{3}{2}} = x^{2 \times \frac{3}{2}}$
 $= x^3$

e $(x^3)^{\frac{5}{3}} = x^{3 \times \frac{5}{3}}$
 $= x^5$

f $3x^{0.5} \times 4x^{-0.5} = 12x^{0.5+(-0.5)}$
 $= 12x^0$
 $= 12$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}} = 3x^{\frac{2}{3}-\frac{1}{6}}$
 $= 3x^{\frac{1}{2}}$

h $5x^{\frac{7}{5}} \div x^{\frac{2}{5}} = 5x^{\frac{7}{5}-\frac{2}{5}}$
 $= 5x$

i $3x^4 \times 2x^{-5} = 6x^{4+(-5)}$
 $= 6x^{-1}$

j $\sqrt{x} \times \sqrt[3]{x} = x^{\frac{1}{2}} \times x^{\frac{1}{3}}$
 $= x^{\frac{1}{2}+\frac{1}{3}}$
 $= x^{\frac{5}{6}}$
 $= (\sqrt[6]{x})^5$

k $(\sqrt{x})^3 \times (\sqrt[3]{x})^4 = x^{\frac{3}{2}} \times x^{\frac{4}{3}}$
 $= x^{\frac{3}{2}+\frac{4}{3}}$
 $= x^{\frac{17}{6}}$
 $= (\sqrt[6]{x})^{17}$

l $\frac{(\sqrt[3]{x})^2}{\sqrt{x}} = x^{\frac{2}{3}} \div x^{\frac{1}{2}}$
 $= x^{\frac{2}{3}-\frac{1}{2}}$
 $= x^{\frac{1}{6}}$
 $= \sqrt[6]{x}$

2 a $25^{\frac{1}{2}} = \sqrt{25}$
 $= 5$

b $81^{\frac{3}{2}} = (\sqrt{81})^3$
 $= 9^3$
 $= 729$

c $27^{\frac{1}{3}} = \sqrt[3]{27}$
 $= 3$

d $4^{-2} = \frac{1}{4^2}$
 $= \frac{1}{16}$

e $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}}$
 $= \frac{1}{\sqrt{9}}$
 $= \frac{1}{3}$

f $(-5)^{-3} = \frac{1}{(-5)^3}$
 $= -\frac{1}{125}$

g $\left(\frac{3}{4}\right)^0 = 1$

h $1296^{\frac{3}{4}} = (\sqrt[4]{1296})^3$
 $= 6^3$
 $= 216$

$$\begin{aligned} 2 \text{ i } \left(\frac{25}{16}\right)^{\frac{3}{2}} &= \frac{(\sqrt{25})^3}{(\sqrt{16})^3} \\ &= \pm \frac{5^3}{4^3} \\ &= \pm \frac{125}{64} \end{aligned}$$

$$\begin{aligned} \text{j } \left(\frac{27}{8}\right)^{\frac{2}{3}} &= \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2} \\ &= \frac{3^2}{2^2} \\ &= \frac{9}{4} \end{aligned}$$

$$\begin{aligned} \text{k } \left(\frac{6}{5}\right)^{-1} &= \left(\frac{5}{6}\right)^1 \\ &= \frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{l } \left(\frac{343}{512}\right)^{-\frac{2}{3}} &= \frac{(\sqrt[3]{512})^2}{(\sqrt[3]{343})^2} \\ &= \frac{8^2}{7^2} \\ &= \frac{64}{49} \end{aligned}$$

$$\begin{aligned} 3 \text{ a } (64x^{10})^{\frac{1}{2}} &= \sqrt{64x^{10 \times \frac{1}{2}}} \\ &= 8x^5 \end{aligned}$$

$$\begin{aligned} \text{b } \frac{5x^3 - 2x^2}{x^5} &= \frac{5x^3}{x^5} - \frac{2x^2}{x^5} \\ &= 5 \times x^{3-5} - 2 \times x^{2-5} \\ &= 5x^{-2} - 2x^{-3} \\ &= \frac{5}{x^2} - \frac{2}{x^3} \end{aligned}$$

$$\begin{aligned} \text{c } (125x^{12})^{\frac{1}{3}} &= \sqrt[3]{125x^{12 \times \frac{1}{3}}} \\ &= 5x^4 \end{aligned}$$

$$\begin{aligned} 3 \text{ d } \frac{x+4x^3}{x^3} &= \frac{x}{x^3} + \frac{4x^3}{x^3} \\ &= x^{1-3} + 4 \times x^{3-3} \\ &= x^{-2} + 4x^0 \\ &= \frac{1}{x^2} + 4 \end{aligned}$$

$$\begin{aligned} \text{e } \frac{2x+x^2}{x^4} &= \frac{2x}{x^4} + \frac{x^2}{x^4} \\ &= 2 \times x^{1-4} + x^{2-4} \\ &= 2x^{-3} + x^{-2} \\ &= \frac{2}{x^3} + \frac{1}{x^2} \end{aligned}$$

$$\begin{aligned} \text{f } \left(\frac{4}{9}x^4\right)^{\frac{3}{2}} &= \left(\sqrt{\frac{4}{9}}\right)^3 x^{4 \times \frac{3}{2}} \\ &= \frac{8}{27}x^6 \end{aligned}$$

$$\begin{aligned} \text{g } \frac{9x^2 - 15x^5}{3x^3} &= \frac{9x^2}{3x^3} - \frac{15x^5}{3x^3} \\ &= 3 \times x^{2-3} - 5 \times x^{5-3} \\ &= 3x^{-1} - 5x^2 \\ &= \frac{3}{x} - 5x^2 \end{aligned}$$

$$\begin{aligned} \text{h } \frac{5x+3x^2}{15x^3} &= \frac{5x}{15x^3} + \frac{3x^2}{15x^3} \\ &= \frac{1}{3} \times x^{1-3} + \frac{1}{5} \times x^{2-3} \\ &= \frac{1}{3}x^{-2} + \frac{1}{5}x^{-1} \\ &= \frac{1}{3x^2} + \frac{1}{5x} \end{aligned}$$

$$\begin{aligned} 4 \text{ a } 81^{\frac{1}{4}} &= \sqrt[4]{81} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{b } x(2x^{\frac{1}{3}})^4 &= x \times 2^4 x^{\frac{1}{3} \times 4} \\ &= 2^4 x^{\frac{4}{3}+1} \\ &= 16x^{\frac{7}{3}} \end{aligned}$$

$$\begin{aligned}
 \mathbf{5\ a} \quad y^{\frac{1}{3}} &= \left(\frac{1}{8}x^3\right)^{\frac{1}{3}} \\
 &= \frac{1}{\sqrt[3]{8}}x^{3 \times \frac{1}{3}} \\
 &= \frac{x}{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \frac{1}{2}y^{-2} &= \frac{1}{2}\left(\frac{1}{8}x^3\right)^{-2} \\
 &= \frac{1}{2} \times \frac{1}{\left(\frac{1}{8}x^3\right)^2} \\
 &= \frac{1}{2} \times \frac{64}{x^{3 \times 2}} \\
 &= \frac{32}{x^6} \\
 &= 32x^{-6}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6\ a} \quad x^{\frac{1}{2}} &= 7 \\
 \left(x^{\frac{1}{2}}\right)^2 &= 7^2 \\
 x &= 49
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad y^{\frac{4}{3}} &= 81 \\
 \left(y^{\frac{4}{3}}\right)^{\frac{3}{4}} &= 81^{\frac{3}{4}} \\
 y &= 27
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad x^{\frac{3}{2}} &= 8 \\
 \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} &= 8^{\frac{-2}{3}} \\
 x &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad z^{\frac{-3}{4}} &= 1000 \\
 \left(z^{\frac{-3}{4}}\right)^{\frac{4}{3}} &= 1000^{\frac{-4}{3}} \\
 z &= \frac{1}{10000}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad 27\sqrt{x} &= \frac{1}{x} \\
 x^{\frac{3}{2}} &= \frac{1}{27} \\
 \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} &= \left(\frac{1}{27}\right)^{\frac{2}{3}} \\
 x &= \frac{1}{9}
 \end{aligned}$$